

**The Preparation Gap:  
Teacher Education for  
Middle School Mathematics  
in Six Countries**

**Mathematics Teaching in the 21st Century (MT21)**



# **The Preparation Gap: Teacher Education for Middle School Mathematics in Six Countries**

**(MT21 Report)**

Prepared by:

William H. Schmidt, Maria Teresa Tatto, Kiril Bankov, Sigrid Blömeke,  
Tenoch Cedillo, Leland Cogan, Shin Il Han, Richard Houang, Feng Jui Hsieh,  
Lynn Paine, Marcella Santillan and John Schwille.

---

This material is based upon work supported by the National Science Foundation under grant No.0231886. Any opinions, findings, and conclusions or recommendations expressed in this materials are those of the authors and do not necessarily reflect the views of the National Science Foundation.

© 2007 MSU Center for Research in Mathematics and Science Education.

Additional copies of this report can be downloaded at <http://usteds.msu.edu>.

The authors gratefully acknowledge the following: Neelam Kher, Kathy Wight, and David Wiley for their careful comments on the draft and in particular, Curtis McKnight who provided valuable assistance and comments under time pressure; Zhiwen Zou (Jimmy) and Sungworn Ngudgratoke for assistance in analyses for the report; Adam Maier for assistance with the literature search; Jacqueline Babcock for organizing the research group meetings and assuring the project ran smoothly; Anil Talwalker for careful attention to detail in word processing; Adrian Beavis and Lawrence Ingvarson for contributions in the development of the Future Teacher Questionnaire; Hans Wagemaker and IEA for providing funding to allow Adrian, Lawrence and other consultants to attend several early planning meetings; Kathy DeAngelo for graphic design; Ike Iyioke for writing the press release; and Susan Pettit Riley for media relations and facilitation of the report production.

# TABLE OF CONTENTS

|   |    |
|---|----|
| Executive Summary .....   | 1  |
| Chapter I: What are the Key Findings? .....   | 3  |
| Chapter II: Theoretical Framework.....  | 10 |
| Chapter III: Study Description .....  | 17 |
| Chapter IV: Knowledge of Mathematics and Mathematics Pedagogy.....  | 22 |
| Chapter V: What are the Course Taking Experiences of Future Teachers<br>in Each of the Six Countries? ..... | 28 |
| Chapter VI: Beliefs about Mathematics .....   | 34 |
| Chapter VII: What are the Policy Implications?.....   | 41 |
| References .....  | 43 |

## List of Tables and Figures

|  |    |
|--|----|
| Table 2.1: Theoretical Framework for MT21 .....  | 11 |
| Table 2.2: Definition of Professional Requirements for Teachers of Mathematics.....  | 13 |
| Table 3.1: Sample Sizes.....   | 19 |
| Figure 4.1: Percentile Distribution of Mathematics Knowledge Scale Scores Across Students<br>Within Each Country.....                    | 23 |
| Figure 4.2: Percentile Distribution of Mathematics Pedagogical Knowledge Scale Scores Across<br>Students Within Each Country .....       | 24 |
| Figure 4.3: Mean Level Performance on the Mathematics Knowledge Scale Scores<br>Across the Six Countries.....                            | 25 |
| Figure 4.4: Mean Level Performance on the Mathematics Pedagogical Scale Scores<br>Across the Six Countries.....                          | 26 |
| Figure 5.1: Mean Percent of the Topics Covered in the Preparation Program<br>in Each Country .....                                       | 29 |
| Figure 5.2: Mean Rating of the Extent of Coverage of Practical Pedagogy in Each Country.....   | 31 |
| Figure 6.1: Mean Rating of the Degree of Belief About the Nature of Mathematics<br>Across the Six Countries.....                         | 34 |
| Figure 6.2: Mean Belief in the Extent of Emphasis That Should be Given to<br>Various Objectives for the Six Countries .....              | 36 |
| Figure 6.3: Mean Rating of the Degree of Belief in Different Ways That Students<br>Learn Mathematics Across the Six Countries .....      | 38 |
| Figure 6.4: Mean Degree of Belief That Variation in Mathematics Achievement is<br>Related to Various Factors for the Six Countries ..... | 40 |



## EXECUTIVE SUMMARY

MT21 is a cross-national study of the preparation of middle school mathematics teachers. Countries participating included Chinese Taipei (Taiwan), South Korea (Korea), Bulgaria, Germany, Mexico and the United States. The preparation of middle school teachers in the US is accomplished through three routes: (a) elementary programs, (b) secondary programs or (c) middle school programs. Data were collected from teachers in their last year of preparation by sampling institutions in each country. Future teachers were questioned on their (1) backgrounds, (2) course taking and other program activities, (3) knowledge relevant to their teaching – mathematical and pedagogical, and (4) beliefs and perspectives on content and pedagogy.

Ten years ago a report was issued which showed that the United States middle school students performed below the international mean on a test of mathematics among some 40 countries. Additionally, the Third International Mathematics and Science Study (TIMSS) indicated that one of the major factors related to that low performance was a middle school curriculum in the US that was unfocused, lacking coherence, and not particularly demanding or rigorous. The study also showed that the top-achieving countries had the opposite type of curriculum. Many states and districts in response have recently increased the rigor of their standards as well as their focus and coherence. The important question then becomes– how shall we prepare our future teachers to teach a more rigorous curriculum to all students? The MT21 project was designed to answer that question by examining how other countries prepared their middle school teachers.

The results clearly suggest that teacher education as defined by the learning opportunities provided likely has an impact on what future teachers know and believe as they leave their teacher preparation program. The differences across the countries, combined with the differences across the three programs of preparation within the United States, all point in that direction.

Mathematical knowledge among future teachers as measured in MT21 was highest in Taiwan and Korea on all five areas of mathematics – algebra, functions, number, geometry and statistics –while the US performance lagged behind scoring anywhere from the middle of the six countries (in statistics) to almost three fourths of a standard deviation below the international mean in functions.

The answer to the question of how to best prepare middle school teachers is more complex than might be expected given these achievement results. The obvious solution of having US future teachers of middle school take more mathematics appears to be the answer but it is only part of the answer. Both Korea and Taiwan whose middle school students have performed well in previous international comparative studies such as TIMSS demanded a different level of preparation on the part of their future teachers than was provided in the United States. In Taiwan and Korea, the level of mathematics preparation was very strong and, in both countries, the amount of emphasis given to the practical issues of mathematics pedagogy was also extensive. In general pedagogy, there was a difference between the two top performing countries. In Taiwan, there was also extensive emphasis given to

the practical aspects of general pedagogy such as classroom management, which was not the case for Korean future teachers. Apart from that difference, it is clear that the future teachers in those two countries have extensive coverage of two of the areas (and in the case of Taiwan all three areas). In none of the six countries was the preparation of teachers done without at least some level of coverage associated with each of the three broad areas.

The contrast of the German profile to that of the future teachers in the US is interesting. The amount of mathematics taken by the future teachers in the US was lower in algebra and analysis than was the case for German future teachers, but the amount of opportunity that was provided in the pedagogy areas was much larger. On the international TIMSS test, eighth graders in both Germany and US had comparable low levels of performance.

The differences in achievement between the Asian eighth graders and the US eighth graders is likely related not only to the “curriculum gap” found in TIMSS but also to a “preparation gap” - the fact that teachers in those countries had a very different configuration of learning experiences as a part of their teacher preparation.

The belief held by some that the preparation of future teachers might be done without any preparation in practical pedagogy seems unwise and should certainly be reconsidered. The fact that none of the five other countries prepare their teachers in this way tells us something. The real question then is not whether such experiences are necessary but rather the nature and the extent of the learning opportunities in each of the three areas that should be available for future teachers. It is quite revealing that the countries whose middle school students continuously perform well on the international benchmark tests have a coherent, focused and rigorous curriculum as well as teachers who have been trained with extensive educational opportunities in mathematics and in the practical aspects of teaching mathematics to students in the middle grades.

# I. WHAT ARE THE KEY FINDINGS?

Mathematics Teaching in the 21<sup>st</sup> century (MT21) is a study of the preparation of middle school mathematics teachers in six countries. Why is such a study needed? What did it find? What led to it?

A report was issued ten years ago that showed United States middle school students performed below the international mean on a test of mathematics among some 40 countries. While US students did reasonably well in earlier grades, a precipitous drop in US students' performance began in middle school. Further, the Third International Mathematics and Science Study (TIMSS) indicated that one of the major factors related to that low performance was the US middle school mathematics curriculum in place at that time.

That mathematics curriculum was characterized as unfocused, lacking coherence and not particularly demanding or rigorous. The phrase, “a mile wide and an inch deep” came to represent the US middle school mathematics curriculum. That was because the curriculum didn't seem to go anywhere, kept covering the same topics repeatedly and as a result covered many topics at each grade level without focus on fewer key topics. Top achieving countries were shown to have a very different type of curriculum — one that was focused, coherent and demanding. In much of the rest of the world, but especially in those top achieving countries, middle school focused on algebra and geometry and not on arithmetic. In contrast, arithmetic was and remains the focus of the eighth grade curriculum for most US students.

What caused this weaker curriculum in middle school mathematics? One conjecture was that the US curriculum didn't focus on more challenging mathematics because US middle school teachers didn't have the background to teach such a curriculum. The obvious question then is: “How did other countries prepare their middle school teachers to be able to teach a more demanding and rigorous curriculum?” This question has become even more urgent as states and districts have recently increased the rigor of their standards as well as their focus and coherence. The pressing question for the US is, “How shall we prepare our future teachers to competently teach a more rigorous curriculum to all students?”

The MT21 project was designed to answer that question by examining how other countries prepared their middle school teachers. Six countries participated including Chinese Taipei (Taiwan), South Korea (Korea), Bulgaria, Germany, Mexico, and the United States. Purposeful samples of institutions were drawn from each of the countries so as to obtain as representative a sample as possible. Data were then collected on the backgrounds, course experiences and beliefs about teaching and learning mathematics on a sample of future teachers within each institution. The course experiences included those in mathematics and those related to the practical aspects of classroom instruction, that is, pedagogy. In addition, a test of mathematics knowledge was administered to those being prepared as teachers. This section focuses on the key findings that resulted from that study of 2,627 future teachers in six countries.



The answer to the basic question driving this study proved more complex than was originally anticipated. The obvious solution of having future teachers of middle school take more mathematics appears to be only part of the answer. Both Taiwan and Korea had students that performed extremely well on international tests such as TIMSS and PISA. The future teachers in Korean and Taiwanese institutions achieved the highest scores on the mathematics tests administered in MT21. They have, in general, more demanding and extensive teacher preparation programs when compared to the US — not just in mathematics, but in pedagogy as well.

The differences in achievement between Asian eighth graders and US eighth graders is likely related not only to the “curriculum gap” found in TIMSS but also to a “preparation gap,” that is, to the fact that teachers in those countries had a very different set of learning experiences as a part of their teacher preparation.

Which learning experiences matter most in preparing teachers to teach challenging mathematics? What should the balance be across the three areas of mathematics, mathematics education and general pedagogy? These are most important questions to be addressed especially given the limitation of time associated with typical university preparation.

None of the six countries left out any of the three areas in teacher preparation but the mix of the three differed and became the defining issue. For example, in Taiwan future teachers were given extensive opportunities in all three areas. In Germany future teachers were given only moderate opportunities in mathematics and little or no opportunity on the practical aspects related either to general or mathematical pedagogy. Mexico, on the other hand, focused their course experiences heavily on pedagogy with little course taking in mathematics. The US is best characterized as having little opportunity for mathematics content and modest opportunities in practical pedagogy.

Against this background of country differences in teacher preparation, there were additionally differences within the US in the ways future teachers of middle school mathematics were prepared. Three distinct programs are present in American universities for preparing future middle school mathematics teachers (this study did not look at alternative certification programs). In the first approach the teachers are prepared through an elementary education program.

A second common type of preparation program involves middle school teachers being trained in a secondary education program so that they become certified to teach from sixth or seventh grade through twelfth grade. Finally, in about one half of the states, there are teacher preparation programs focused specifically on the middle school.

There were substantial differences in the preparation of the future teachers across the three types of programs in the opportunities associated with each of the three areas. We also found differences in what future teachers knew about mathematics as demonstrated on the MT21 test, as well as what they believed about the teaching of mathematics. It is likely that what learning opportunities they have will influence what happens when these future teachers enter the classroom to teach mathematics. However, this is just a conjecture — the definitive study of the relationship of teacher preparation to classroom instruction and student achievement has yet to be done.

In general, we found that US future teachers prepared through secondary programs had a better knowledge of mathematics than those prepared in either of the other program types. Not surprisingly, their preparation in mathematics was more demanding and rigorous as they were typically required to take a larger number of advanced mathematics courses. Their experiences related to the two areas of pedagogy typically were weaker than was the case for those teachers prepared through the elementary program. By contrast, in Taiwan, where all future teachers of middle school mathematics are prepared in a secondary type program, the demands were extensive both in mathematics and the two areas of pedagogy.

The results presented in this report suggest an important hypothesis that needs further study. The differences between the countries in the nature of teacher preparation, with the concomitant differences in what mathematics those future teachers know — and combined with such differences between the different teacher education programs within the United States — suggests that “teacher education matters.” It seems clear that one must challenge the idea that anyone who has a degree in mathematics can teach middle school without any background in pedagogy, an idea suggested by some policy-makers. No such approach for preparing middle school mathematics teachers existed in any of the six countries or in any of the 34 institutions studied. The real issue is the balance among the three areas of mathematics, mathematics pedagogy and general pedagogy. Our goal is for MT21 to make a modest contribution in addressing that issue. The following summarizes some of the key findings.

## ■ ACHIEVEMENT

- The mathematics knowledge of US future middle school mathematics teachers generally is very weak compared to future teachers in Taiwan and Korea. It is also weak compared to German future teachers in all areas except statistics.
- Taiwanese and Korean future teachers were the top performers in all five areas of mathematics knowledge — including algebra, functions, number, geometry and statistics.
- School algebra (which includes functions) is the topic that, across almost fifty countries studied in TIMSS, was the major focus of instruction at seventh and eighth grade.
- On the algebra and functions tests, US future teachers performed at or near the bottom among the six countries — over a full standard deviation below the performance of future Taiwanese teachers.
- The results for the statistics test were the only bright spot in future US teachers’ performance. The US future teachers scored near the mean of the six countries.
- Future middle school teachers prepared by a secondary program performed somewhere between one-half to three-fourths of a standard deviation higher in algebra, functions, geometry and number compared to those prepared in either of the other two programs. The difference was slightly less in statistics.
- Future US middle school teachers prepared through a secondary program were slightly below the international mean for algebra, functions and geometry and slightly above it for number.

That performance was still somewhere from one-half to almost a full standard deviation below that of Taiwan. On the statistics test, US future teachers prepared in a secondary program scored about a third of a standard deviation above the international mean, essentially tying them with Taiwan.

The differences in mathematics knowledge among the US teacher preparation programs do not support the inference that one program is better than another and causes differences in achievement. The difference could as easily be related, at least in part, to what type of student chooses which type of program.

## ■ COURSE TAKING

### Mathematics

- On average, the Taiwanese and Korean future teachers reported taking courses that covered around eighty percent or more of the advanced mathematics topics typically covered in undergraduate mathematics programs.
- For analysis (the study of functions) Taiwanese future teachers covered virtually all of the topics (ninety-six percent) while, in Korea, the coverage was seventy-nine percent.
- In the algebra and analysis courses which provide the mathematical background for middle school algebra, the Taiwanese, Korean and Bulgarian future teachers all covered around eighty percent or more of the possible topics while Germany covered around 60 to 70 percent.
- Mexican and US future teachers covered less than half of the analysis topics. The same was true for Mexico on the algebra topics, but US future teachers covered on average 56 percent of those topics.
- US future teachers received a very different education in mathematics than was the case in Taiwan, Korea, Bulgaria and Germany.
- Future middle school teachers in the US trained by the three different programs had quite different course-taking experiences. This was consistent with the differences found in mathematics knowledge. Students prepared by the secondary program took anywhere from 20 to 35 percent more of the advanced topics than did the future teachers trained through either of the other two programs.

### Pedagogy

For mathematics pedagogy and general pedagogy, the focus was not on the coverage of formal disciplines which provide the intellectual underpinnings for the field of education (such as educational psychology, sociology of education, child development or the theory and history of mathematics education) but rather on the practical aspects of what is involved in the actual teaching of middle school mathematics.

- The variation across countries on the extent of coverage in these areas differed from that reported for mathematics. For example, Mexico and the US provided almost as much or more coverage especially in the area of general pedagogy.
- For practical general pedagogy, Taiwan and Mexico provided strong coverage. The US provided more modest coverage while Germany essentially provided little or no such coverage at least as reported by the future teachers.
- In mathematics pedagogy similarly, Taiwanese and Mexican future teachers had strong coverage in their teacher preparation. On the other hand, German and US future teachers reported having modest or little coverage.

A separate area of mathematics education more related to mathematics than pedagogy focuses on the coverage of the advanced mathematics that provides the background for topics found in the middle school curriculum. Some mathematicians have called this an “advanced treatment of elementary topics.” It involves the study of mathematics but specifically as it relates to the elementary topics that will be found in school mathematics.

- Taiwanese and Korean future teachers reported extensive coverage for this area. The mean value was higher than that for mathematics pedagogy and general pedagogy in both countries.
- US future teachers indicated only modest coverage, as was the case for German future teachers.

## General Observations

Taiwan and Korea have typically performed well in international tests such as TIMSS and PISA. In fact, both of these countries were among the highest performing in TIMSS. Consequently, we have chosen to use their pattern of future teacher course experiences as an international benchmark.

- By those standards, US future teachers did not have the same level of advanced mathematics preparation as the future teachers of those two countries. In fact, by those standards, the reported US course taking was extremely low in mathematics – future teachers studied only 43 percent of the advanced topics compared to the 79 to 86 percent studied by Taiwanese and Korean future teachers.
- In pedagogy, the extent of coverage was again substantially less than that provided by the two Asian countries, but was not as low as in the case of mathematics. (The only exception was for general pedagogy where the Korean mean was lower than that for both Taiwan and the US.)
- Correspondingly, the performance of US future teachers on the analysis (functions) test was one and one-half standard deviations below the Taiwanese future teachers’ performance and more than a standard deviation below that of the Korean future teachers’ performance. For algebra, US performance was well over a standard deviation below that of the performance of the future teachers in both Taiwan and Korea. Differences in terms of performance on the

other three areas (number, geometry and statistics) were smaller but ranged anywhere from one-third of a standard deviation to over one standard deviation below the Asian countries.

- A US policy issue in need of examination is suggested from the data: future teachers prepared in the middle school program end up possibly with the worst of both worlds in terms of learning opportunities. Their training in mathematics is not different from that of the training provided to future middle school teachers in the elementary program but it is less by a sizeable amount (20 to 35 percent fewer advanced topics covered) than typical middle school future teachers' prepared in the secondary program.
- They also tend to get less practical pedagogical training than those teachers trained in an elementary program.

## ■ BELIEFS ABOUT MATHEMATICS

### The Nature of Mathematics

- Beliefs about the nature of mathematics are very similar across the six countries but especially so for Korea, Taiwan and Bulgaria where future teachers indicated moderate to strong agreement that mathematics is formal and algorithmic as well as creative and useful.
- The only significant difference among the three preparation programs in the US on this dimension is that those future teachers prepared in the elementary program but especially in the middle school program more strongly agreed that mathematics is algorithmic than those future teachers prepared in a secondary program.

### What to Teach?

- When the future teachers were asked to indicate the extent of coverage they would give to each of a set of objectives for middle school instruction, the future teachers in all six countries indicated moderate to extensive emphasis should be given to all objectives. The suggested emphasis was among the lowest in each country for objectives related to communicating mathematical ideas. In no case did typical future teachers indicate little emphasis for any of the objectives.
- The affective objectives such as having students become interested in and enjoying mathematics received the greatest degree of emphasis in all countries.
- Future US teachers indicated a more moderate amount of emphasis should be given to teaching mathematics skills, which was consistent with Germany and Korea but not with any of the other countries. US future teachers also indicated more emphasis on the communication objectives than did the future teachers in the other five countries.

## The Way Students Learn

- Few differences were found among the six countries' future teachers on beliefs about the way students learn. There appears to be general agreement on how students learn mathematics.

## Who Can Learn Mathematics

- This area of beliefs focused on why there is large variation in mathematics achievement. The main hypotheses posed to the future teachers about why there is this variation included: differences in natural ability, gender or ethnicity differences (categorical differences) or developmental differences.
- Large cross-national differences existed. The typical Taiwanese future teacher leaned slightly in the direction of believing all three were important. In Korea, they were similar in their beliefs but indicated a moderate disagreement with the categorical explanation.
- The US was more like Germany and Mexico, strongly rejecting the natural ability and categorical difference explanations and moderately to strongly agreeing with the notion of developmental differences.

## US Program Comparisons

- On the nature of mathematics, the main program difference, which was statistically significant, indicated that those teachers prepared under middle school and elementary programs agreed more strongly with the fact that mathematics is algorithmic.
- There were no program differences related to the importance of middle school objectives.
- On the beliefs of future teachers regarding an explanation of the variation in mathematics achievement, there were no differences between the programs in beliefs about the importance of categorical or developmental differences. Future teachers prepared in a middle school program believed that natural ability is more significantly related to achievement differences than did the teachers prepared in an elementary program.

## II. THEORETICAL FRAMEWORK

TIMSS and PISA revealed extensive insight into the conditions for learning successes and failures of students in elementary and middle school. What is presently lacking is insight into the quality of their teachers. Several questions follow from this:

- What does the professional knowledge look like for these teachers and what beliefs do they hold?
- What is the effect of their teacher education?
- Which characteristics of teacher education are relevant for the professional competence of future teachers?

These issues were analyzed systematically and by international comparison for the first time for future middle school mathematics teachers. MT21 studies a group of future teachers that will be confronted with a central role in the preparation of future generations for an internationally driven and information-based economy. Mathematics not only belongs to the core academic subjects worldwide (Mullis et al., 2004: 365); to have mathematical competence is also a central need for meeting everyday occupational requirements (Freudenthal, 1983). Mathematical competence provides a tool for problem solving and is required to understand the world.

### ■ THE MODEL

At its deepest level, the approach of MT21 is to devise a conceptualization of professional competence for future teachers that is suitable to collecting an internationally comparable set of empirical data and relating this to characteristics of teacher preparation programs. Instruments were developed for all levels of analysis that allowed for international comparisons. Based on this, MT21 for the first time provides empirical results for the professional knowledge and beliefs of future middle school (internationally: lower secondary) mathematics teachers to help understand the effect of teacher education in mathematics. The following table provides an overview of the underlying model for MT21 (Blömeke et al., in print). In the present report, data are presented relative only to some of the concepts at the micro and intermediate levels. Chapter IV, for example, focuses on the individual future teachers' competence while Chapter V reports results relative to future teachers' perceptions on the institutionally implemented curriculum. Future publications will report on data relative to all other concepts found in Table 2.1.

**Table 2.1: Theoretical Framework for MT21**

|                           |           |   |  |  |   |                                  |                 |
|---------------------------|-----------|---|--|--|---|----------------------------------|-----------------|
| <b>Macro level</b>        | Level III | <b>Cultural context</b>                           |  |  |   |                                  |                 |
|                           |           | Rationale of the society                          | Social status of the teaching profession |  | Social status of mathematics                  |                                  |                 |
|                           | Level II  | <b>Education system</b>                           |  |  |   |                                  |                 |
|                           |           | Rationale of the education system                 |  | Goals of schooling                                   | Working conditions of teachers                |                                  |                 |
|                           | Level I   | <b>Teacher-education system</b>                   |  |  |   |                                  |                 |
|                           |           | Goals of teacher education                        | Content structure                        | Institutional structure                              | Relationship of theory and practice           | Selectivity                      |                 |
| <b>Intermediate level</b> | Level II  | <b>Institutionally intended curriculum</b>        |  |  |   |                                  |                 |
|                           |           | Learning goals and content                        | Teaching methods                         | Accountability                                       | Academic advising                             | Selectivity                      |                 |
|                           | Level I   | <b>Teacher educators</b>                          |  |  | <b>Institutionally implemented curriculum</b> |                                  |                 |
|                           |           | Knowledge   | Beliefs                                  | Demographics   | Learning goals/content                        | Teaching methods                 | Selectivity     |
|                           |           | Intended learning goals                           | Intended teaching methods                |  | Accountability                                | Academic advising                | FTs composition |
| <b>Micro level</b>        | Level II  | <b>Individual Future Teacher's preconceptions</b> |  | <b>Individual Future Teacher's use of curriculum</b> |   |                                  |                 |
|                           |           | Knowledge   | Beliefs                                  | Perception of learning goals and use of content      |   | Experience with teaching methods |                 |
|                           |           | Personality                                       | Demographics                             | Accountability                                       |   | Academic advising                | Selectivity     |
|                           |           |   |  | Amount of learning time                              |   | Learning strategies              | Emotions        |
|                           | Level I   | <b>Individual Future Teacher's competencies</b>   |  |  |   |                                  |                 |
|                           |           | Knowledge   | Beliefs                                  |  | Personality characteristics                   | Demographics                     |                 |



Empirical analyses of teacher training until recently have been virtually non-existent (see the following for an overview of related research: Houston, 1990; Tatto, et al., 1993; Sikula, Buttery & Guyton, 1996; Blömeke, 2004; Tatto et al., 2004; Cochran-Smith & Zeichner, 2005; and Scwille & Dembele, 2007). There is also a lack of research on teacher preparation in specific subject matters. Extensive empirical research exists on mathematics instruction (Bishop, 1991; Leung, 1995; Schmidt et al., 1997; Kaiser, 1999; Hiebert et al., 2003) but the current state of research on the qualifications of mathematics teachers and of teacher education in mathematics is lacking (Lerman, 2001; Blömeke, 2004; Adler et al., 2005). Only recently has there been research on the qualifications of mathematics teachers (Ball & Bass, 2003; Hill, 2007; Ferrini-Mundy et al., 2006; Schmidt et al., 2006; Brunner et al., 2006). MT21 uses a completely new approach to examine teacher education in mathematics and to test the knowledge of future teachers.

Analyzing teacher education for international comparisons is a particular challenge. Differences in the structure of training makes acquiring comparable data complicated. The OECD and European Union have, however, in recent years intensified their efforts to record characteristics of teacher education through international comparison. The educational network of the European Union, Eurydice, recorded data about the teaching job market (Eurydice, 2002b, 2003) and about teacher education (Eurydice, 2002a) for the first level of secondary school (middle school in the US). Based on this, the consequences of educational policies were extracted (Eurydice, 2004). The parallel OECD initiative “Attracting, Developing and Retaining Effective Teachers” provides comparable structural information for 25 countries, only seven of which, however, are not located in Europe (OECD 2004b).

In fact, Eurydice and OECD primarily emphasize the variation in educational systems. These can be summarized by two approaches, both represented in MT21. The basic teacher education approach is one in which a specialized field (such as mathematics), specialized pedagogy (such as mathematics education) and general pedagogy are taught in parallel and completed with a Bachelor’s degree and if necessary a Master’s degree. There is also a consecutive two-stage teacher education approach, in which the instruction in specialized pedagogy and general pedagogy follows the first stage. It results in a Bachelors of Science in a specialized field such as mathematics. The second stage concludes either with the acquisition of a teaching license or a Master’s degree. The second stage is often housed in a separate institution, as is the case in Germany.

## ■ PROFESSIONAL COMPETENCE

The central dependent variable of the study is based on the notion of professional competence. MT21 defines this in reference to Weinert (1999, 2001). Table 2.2 lists the requirements for middle school mathematics teachers (see also Bromme, 1992). A and B portray core tasks of teachers that are the only ones able to be assessed in such an international cognitive study and are therefore at the core of MT21. To accomplish these tasks, teachers need cognitive abilities and skills that make up professional knowledge and professional beliefs (Bromme, 1997; Weinert, 2001).

**Table 2.2: Definition of Professional Requirements for Teachers of Mathematics**

(Source: Blömeke, 2005, P. 4)

| Teacher tasks  | Situations  |
|--|---|
| A: Choice of themes, methods; sequencing of learning processes         | 1. Selecting and justifying content of instruction<br>2. Designing and evaluating lessons   |
| B: Assessment of student achievement; counselling of students/ parents | 1. Diagnosing student achievement, learning processes, misconceptions, preconditions<br>2. Assessing students<br>3. Counselling students and parents<br>4. Dealing with errors, giving feedback |
| C: Support of students' social, moral, emotional development           | 1. Establishing teacher-student relationship<br>2. Fostering the development of morals and values<br>3. Dealing with student risks<br>4. Preventing, coping with discipline problems            |
| D: School development  | 1. Initiating, facilitating cooperation<br>2. Understanding of school evaluation  |
| E: Professional ethics   | 1. Accepting the responsibility of a teacher  |

## Professional Knowledge

The cognitive performance of future mathematics teachers can be further divided into mathematics knowledge and mathematics pedagogical knowledge. The first factor is defined by Shulman (1985) more generally as *content knowledge* (see also Baumert & Kunter, 2006; Blömeke, 2002, 2005). The second factor is characterized as *pedagogical content knowledge*.

Since it is a significant component of teaching practice, mathematics pedagogical knowledge is further subdivided:

- *Instructional planning*. This is needed before instruction begins. The mathematics content for students must be chosen appropriately, simplified, and prepared with the use of various representations (Krauthausen & Scherer, 2007; Vollrath, 2001). We refer to this aspect of mathematics pedagogical knowledge as “teaching.”
- *Student learning*. This is knowledge related to the *learning processes* that occur during instruction that focuses on teacher-student interactions. Such knowledge includes classifying student answers, verbal or written, in relation to the tasks or questions that stimulated them, asking questions at different levels of complexity, identifying common misconceptions, providing feedback, and reacting with appropriate intervention strategies. Such knowledge is assigned a prominent position, since it has been found relevant to the attainment of advanced student performance (Blum et al., 2004; Helmke, 2004; Klieme, Schümer & Knoll, 2001).

- *Curricular knowledge.* This focuses on the development of student competence in mathematics in later years of school. What would it mean for later lesson units for example, if a classic subject area of school mathematics in the middle school curriculum were removed or taught as a part of a different sequence.

## Professional Beliefs

An understanding of “competence” as used in MT21, includes, in addition to the cognitive components the instructional, professional and personal beliefs held by the teacher. A special role is attributed to beliefs about what defines mathematics (Leder, Pekkonen & Törner, 2002; Leinhardt & Greeno, 1986; Peterson et al., 1989). If beliefs are operationalized in terms of the subject matter, the correlation of student performance with beliefs was found to be higher than for single cognitive components of professional competence (Bromme, 2005). In this way, the beliefs provide an orienting and action-guiding function (Grigutsch, Raatz & Törner, 1998; Staub & Stern, 2002) that builds a bridge between knowledge and action. They become an important indicator for later teaching and student performance (Brown & Rose, 1995; Nespor, 1987; Short & Short, 1989).

This report differentiates between three groups of beliefs: (Calderhead, 1996; Hofer & Pintrich, 2002; Op’t Eynde, De Corte & Verschaffel, 2002; Thompson, 1992):

- Epistemological beliefs regarding mathematics
- Instructionally related beliefs about teaching
- Instructionally related beliefs of how students learn mathematics

**Epistemological convictions about mathematics:** Epistemological beliefs refer to the structure and origin of knowledge (Hofer & Pintrich, 2002; Schommer, 2002). The epistemological beliefs of future teachers regarding mathematics are considered from two perspectives: (1) from the structural perspective of mathematics as an academic discipline, and (2) from the anthropological perspective of the origin of mathematical competence. The latter deals with the question, “Are mathematical abilities learnable or do they stem primarily from deeply rooted talents?”

A study by Grigutsch et al. (1998) identifies among German mathematics teachers four views as to the nature of mathematics that they also characterize as “mathematical world views”:

- “*Formalism.*” This first view they classify as seeing mathematics as an abstract system that consists of axioms and relations.
- “*Schema Orientation.*” This second view is considering mathematics as an aggregate knowledge of rules, facts and procedures.
- “*Process Orientation.*” The third view is seeing mathematics as an activity that emphasizes the diversity of mathematical problem solving, the meaning of creativity and one’s own initiative in problem solving.

- “Application Orientation.” This fourth view the authors classify as considering mathematics as a useful tool for problem solving in everyday life.

Formalism and Schema Orientation constitute the static aspect of mathematics according to Grigutsch et al. On the other hand, the dynamical aspects of mathematics are defined through the process (creative) and application (useful) orientation. In MT21, a shortened version of the survey by Grigutsch et al. (1998) was used in which the four scales are identified as formal, algorithmic, creative and useful.

**Beliefs About the Teaching and Learning of Mathematics Related to Instruction:** Beliefs about the teaching and learning of mathematics are close to the teaching and learning of mathematics itself. That is, they concretely consider goals of future teachers in mathematics and implementing possible instruction methods, and take into consideration the task of classroom management. *Goals* provide teachers with a higher-order orientation for individual decisions in the course of their instructional planning, execution and reflection. These individual decisions in the broadest sense can be conceived of as *instructional methods* that are referred to in the second dimension above. Beliefs regarding *classroom management* of future mathematics teachers must be considered since implementing goals and methods depends significantly on the extent that the available study time is effectively used.

## ■ CHARACTERISTICS OF TEACHER EDUCATION PROGRAMS

A central goal of MT21 is connecting “professional competence” (a dependent variable) with the characteristics of teacher training. The present state of research regarding opportunities to learn (OTL) for future mathematics teachers suffers from the fact that only crude data exists about the educational components — mathematics, mathematics pedagogy, general pedagogy and practical experience in the school. In many studies, the number of courses taken is used to define OTL. MT21, on the other hand, went further in detail, and as a result, the actual content is recorded more precisely.

In the tradition of other studies of empirical educational research — especially, those of the International Association for the Evaluation of Educational Achievement (IEA) — MT21 distinguishes at the institutional level between intended and implemented characteristics of the teacher training.

For this reason a multifaceted approach was chosen in describing learning opportunities in teacher education. The individual instruments are largely organized in parallel to one another. The facets are: an expert survey about the formally designated requirements, a document analysis of a sample of course offerings, a survey of instructors as mediators of the educational offerings, as well as a survey of the future teachers.

The content of mathematics teacher education is differentiated according to the educational components of mathematics, mathematics pedagogy, general pedagogy and practical experience in the school. The basic principle is to differentiate between the formal discipline and the actual tasks. Finding an internationally comparable and at the same time nationally accurate representation of mathematics pedagogy and general pedagogical components was especially difficult. The content of mathematics could be considered somewhat standardized. A further difficulty arose in meeting the requirements of different institutional forms of practical experience. For example, while Germany assigns this to a separate phase of education, the practical experience in the US is part of the university education.

The collection of topics covered in mathematics is oriented, despite all differences in conceptual detail and subsequently in classification, toward an international definition of university basic courses in the baccalaureate education and are generally obligatory for all students (for example, linear algebra or analysis). Also included are advanced courses that are often designated as electives (for example, topology).

### III. STUDY DESCRIPTION

*The Preparation Gap* is MT21's first report. As mentioned earlier, MT21 is a small-scale international study funded by the National Science Foundation. The goal was to examine how middle school (lower secondary school internationally) mathematics teachers were prepared in the six countries.

This report focuses on an initial set of descriptive results for future teachers in their final year of preparation. Later reports will present more formal and analytical examinations of the data including model fitting. Our goal here is simple — to describe how key variables vary across the six countries. Those variables relate to achievement, opportunity to learn and beliefs.

TIMSS data revealed that countries with higher achievement have teachers who teach substantially different content than that of their less accomplished counterparts (see Schmidt et al., 1996; Schmidt et al., 2001). The 1996 Report of the National Commission on Teaching and America's Future argued that what teachers know and do in the classroom matters for pupil learning. It also argued that teacher education might be a viable policy tool to improve the quality of education (National Commission on Teaching and America's Future, 1996). US reform efforts are consistent with this line of thought. They have introduced standards to measure teacher quality as it relates to student achievement. This, in turn, led to accountability concerns regarding teacher preparation programs (INTASC, 1995; Murray, 2000; Leithwood, Edge & Jantzi, 1999; NCATE 2000).

One of the great values of international (cross-national) comparative research is that it allows each participating country to gain insight through comparisons with other systems and models. These comparisons from other countries can help develop insights on policy that may help to improve education in one's own country. Therefore, the MT21 project sought to use international comparative studies' potential to make clearer important aspects of teacher preparation. MT21, when designed, sought to acknowledge the connection between teacher preparation and subsequent pupil achievement. To do this, the decision was made to conduct the study in a small number of countries that had shown substantial differences in international comparisons of students' mathematics achievement. Early discussions involved researchers from a number of countries (some of these early meetings were supported in part by IEA funding). The final research was conducted in six countries: Bulgaria, Germany, Mexico, South Korea, Taiwan and the United States. In each country, MT21 studied a purposeful sample of teacher-preparation institutions so as to obtain as representative a sample as possible.

There were three routes defining lower secondary teacher preparation across the six countries. The first route — the only one found in Bulgaria, Taiwan and Korea — prepares secondary teachers and that includes the preparation of middle school teachers. The second route focuses directly on specifically preparing middle school teachers. This is the only route among the Mexican institutions studied. The third route is preparing middle school teachers as part of the elementary teacher

preparation program. The German model includes both the secondary and primary routes. The US model includes all three routes.

First, consider the Asian countries — Korea and Taiwan. Both prepare future lower secondary teachers through a secondary program located at universities with a four-year study course. The Taiwanese institutions sampled included both normal (historically, primarily teacher preparation institutions) and regular universities. In the latter case, the future teachers are members of various departments at those universities and affiliated with a center focusing on teaching.

The German approach includes two phases. Phase 1 includes university study in mathematics. Phase 2 follows it. This involves practical studies including teaching, and is conducted at a separate institution and lasts from 18 to 24 months. For Bulgaria, future lower secondary teacher preparation is done at a university and involves four years with students affiliated with the Department of Mathematics.

For the American countries of Mexico and the US, teacher preparation involved four years of education (except for one institution in the US which took five years). In Mexico, future lower secondary teachers are prepared at normal schools (at least among those sampled for this study). In the US, preparation was done at the university level. Students are affiliated with an education school or department.

## ■ SAMPLE

MT21 sampled at the institution level in each country. The goal was surveying all eligible future teachers within each sampled institution. The result was a convenience sample of future teachers in each country. The total number of institutions sampled across the six countries was 34 (see Table 3.1). The sample was not a random sample of all teacher preparation institutions in any of the countries. The goal was to obtain a reasonably representative sample of the whole country including some of the variation found across all teacher preparation institutions in the country.

Six institutions prepare future teachers in Bulgaria. MT21 sampled the three that prepared the largest number of future lower secondary teachers. In Mexico, MT21 chose five institutions to be regionally representative. The Taiwanese sample included five institutions representing a variety of teacher preparation institutions across the entire country. The sampling included the largest and most prestigious university as well as others whose prestige varied and some of which also produced larger numbers of future teachers. All normal universities were included in the sample.

For Korea, MT21 sampled four institutions. As in Taiwan, sampling included institutions with different levels of reputation. The German sample included four regions in which all universities and surrounding Phase 2 institutions were included. The sample cuts across important structural characteristics of Germany. Since Germany is the only country in MT21 that includes two phases, Table 3.1 lists sample sizes for two types of institutions.

**Table 3.1: Sample Sizes**

| <b>Country</b> | <b>Number of Institutions</b> | <b>Number of Future Teachers</b> |
|----------------|-------------------------------|----------------------------------|
| Bulgaria       | 3                             | 161                              |
| Taiwan         | 5                             | 668                              |
| Germany        | 1st-4; 2nd-22                 | 848                              |
| Korea          | 4                             | 210                              |
| Mexico         | 6                             | 358                              |
| United States  | 12                            | 382                              |
| <b>Total</b>   | <b>34</b>                     | <b>2627</b>                      |

The US sample is probably the least representative. The US has over 1300 institutions preparing future teachers. A sample representative of the US would require a sample drawn in a statistically representative manner and would yield many more than the 12 institutions that participated in the MT21 project. Nonetheless, the US institutions were chosen so that they cut across some of the major categories considered important in understanding differences among institutions. Some were large institutions that turn out very large numbers of future teachers. Others were smaller and more selective. Several represented former ‘Normal’ schools that had been created many years ago explicitly for the task of preparing teachers. The sampling resulted in 12 institutions drawn from eight states including California, Texas, Michigan, Arizona, Georgia, Indiana, Minnesota and Ohio.

To make these non-random samples as representative as possible in each of the six countries, MT21 weighted the results. We did this by estimating the number of teachers prepared at each of the sampled institutions. MT21 then obtained an estimate of the total number of lower secondary teachers prepared in each country. Using those estimates, we generated a set of weights to provide estimates from the sampled institutions that were more consistent with the output of each country’s institutions.

The best approach, of course, would have been to have the measures of size available at the outset of the study and use this to draw samples with probabilities proportionate to size. The size and the scope of this research project did not allow that possibility. The weights are a post-hoc attempt to make the results more representative. Strictly speaking, the results should not be thought of in terms of country comparisons as much as a comparison among institutions selected in each of the countries. For the sake of simplicity in presentation and discussion, we use the shorthand of referring to the country such as “South Korea” rather than the more cumbersome but more accurate “institutions sampled from South Korea.”



## ■ INSTRUMENTS

MT21 sought to focus on policy-relevant variation in teacher preparation. Therefore, MT21 sought to measure what individuals learned in their teacher preparation programs. We did this by surveying students' learning opportunities as they are structured and provided by educational institutions and faculties of mathematics, mathematics pedagogy, and general pedagogy. Three main surveys were developed for this purpose:

- (1) Institution Survey. This collected program information including entry requirements, academic course requirements, and program length;
- (2) Faculty Surveys. This included questions about the type of learning activities employed in the courses taught and questions about instructors' beliefs about the learning and teaching of mathematics; and
- (3) Future Teacher Surveys. These are described in more detail below.

The Institution Survey was lengthy and detailed. It gathered program-specific information about entrance requirements, student recruitment, course and practical school experience requirements, typical course topic electives, self-perceived relative program strengths and recent or anticipated reforms.

Four types of Faculty Surveys were developed, each intended for those who taught or supervised future teachers. We were interested in four types of faculty and topic areas — mathematics, mathematics education, general education and school-based practical experiences. Questions in these surveys paralleled a subset of issues included on the Future Teacher Surveys. These issues dealt with such things as perspectives on mathematics and the teaching and learning of mathematics, as well as a few questions characterizing the learning environment and activities future teachers might encounter in courses taught by the responding faculty member.

The Future Teacher Survey had four main parts:

- (1) Demographics and academic background,
- (2) Academic program learning opportunities,
- (3) Beliefs and perspectives on schooling, teaching and learning, and
- (4) Knowledge related to the teaching of middle school mathematics.

All future teachers completed one of two forms. The demographics/background and academic opportunities portions were identical for both forms. The other two areas shared a few common items between the forms but most of the questions were unique to a form for these areas.

MT21 wanted to gather information from two different cohorts of future teachers — those at the very beginning of their program (“Beginners”) and those who were at the end of their program. This was done to estimate what individuals actually gain in their teacher preparation program.

Identifying and comparing those at the end of their program (for example, those who would be ready to teach by the beginning of the next school year) was much easier operationally than to identify those who were just beginning their program.

Definitions for the “beginners” cohort differed across institutions according to program characteristics. Some programs limited enrollment and required students to apply for entry into the program after fulfilling certain pre-requisites, which might require two to three years at the university. Other programs were able to identify students upon their entry to the university. Thus the length of time “beginners” had been at their institution ranged from less than a year to nearly three years. The vast majority of “beginners” in Mexico and Taiwan had about two years of teacher preparation and university experience. Beginners in the Bulgarian, German and South Korean samples had about one year of such experience. The US sample was almost evenly divided among those having about one, two, and three years of experience in their program/ university.

This first report only focuses on results from the Future Teacher Survey and, more specifically, on the surveys completed by those students who were in their final year of preparation. Of these final-year students who completed those surveys, about 75 percent were female in Bulgaria and the US. They comprised a little over 60 percent of the samples from Mexico and South Korea but a little less than half the samples from both Germany and Taiwan.

Students indicated their age according to these categories: 18-21, 22-24, 25-27, 28-30, 31-35, 36-39 and over 40. There were students in each country for each of these age ranges except for the “over 40” category. However, the typical age for students in their final year differed across the six countries. Students in Bulgaria were largely aged 22-24. The majority of the students in Taiwan and the US were also in the 22-24 category although not as predominately as in Bulgaria. Another fifth of the students in Taiwan selected the next older category, 25-27, while only about a tenth of US students selected it. Nearly a fifth of US students selected the 18-21 category. About a third of the final year students in Mexico selected the 18-21 category while less than a half selected the 22-24 category. Less than half of these students in South Korea selected the 22-24 category, but another third chose the 25-27 range. Only a third of the final year German students selected the 25-27 category, with about a fourth of these students selecting each of the next two categories, 28-30 and 31-35. Future MT21 reports will deal with other aspects of the study including characteristics of the faculty and institutions as well as achievement results related to general pedagogy.

## IV. KNOWLEDGE OF MATHEMATICS AND MATHEMATICS PEDAGOGY

MT21 developed eight achievement scales from the test items administered to the future teachers. The focus of the achievement items was on the mathematics underlying topics typically taught at the lower and upper secondary level across some 40 countries including the six involved in MT21. The curriculum analysis done as a part of TIMSS (Schmidt et al., 2001) was used to identify the topics. These included algebra, functions, number, geometry and statistics. The MT21 mathematics items were not designed to be at an advanced level associated with an undergraduate mathematics degree. Rather, they were at an advanced level of the mathematics necessary for a deep understanding of the topics taught in a challenging, rigorous and coherent middle and high school mathematics curriculum (Valverde & Schmidt, 2000).

### ■ SCALE DEVELOPMENT

Five scales focused directly on mathematics knowledge related to number, geometry, algebra, functions and statistics. These had 36, 14, 11, 19 and 12 items, respectively. In addition, of these 92 items (score points), 59 also focused on the teaching or pedagogical aspects of mathematics in each of the five areas. The three scales focused on the pedagogical knowledge related to curriculum (18 items), instruction or teaching (18 items) and the student as learner (23 items).

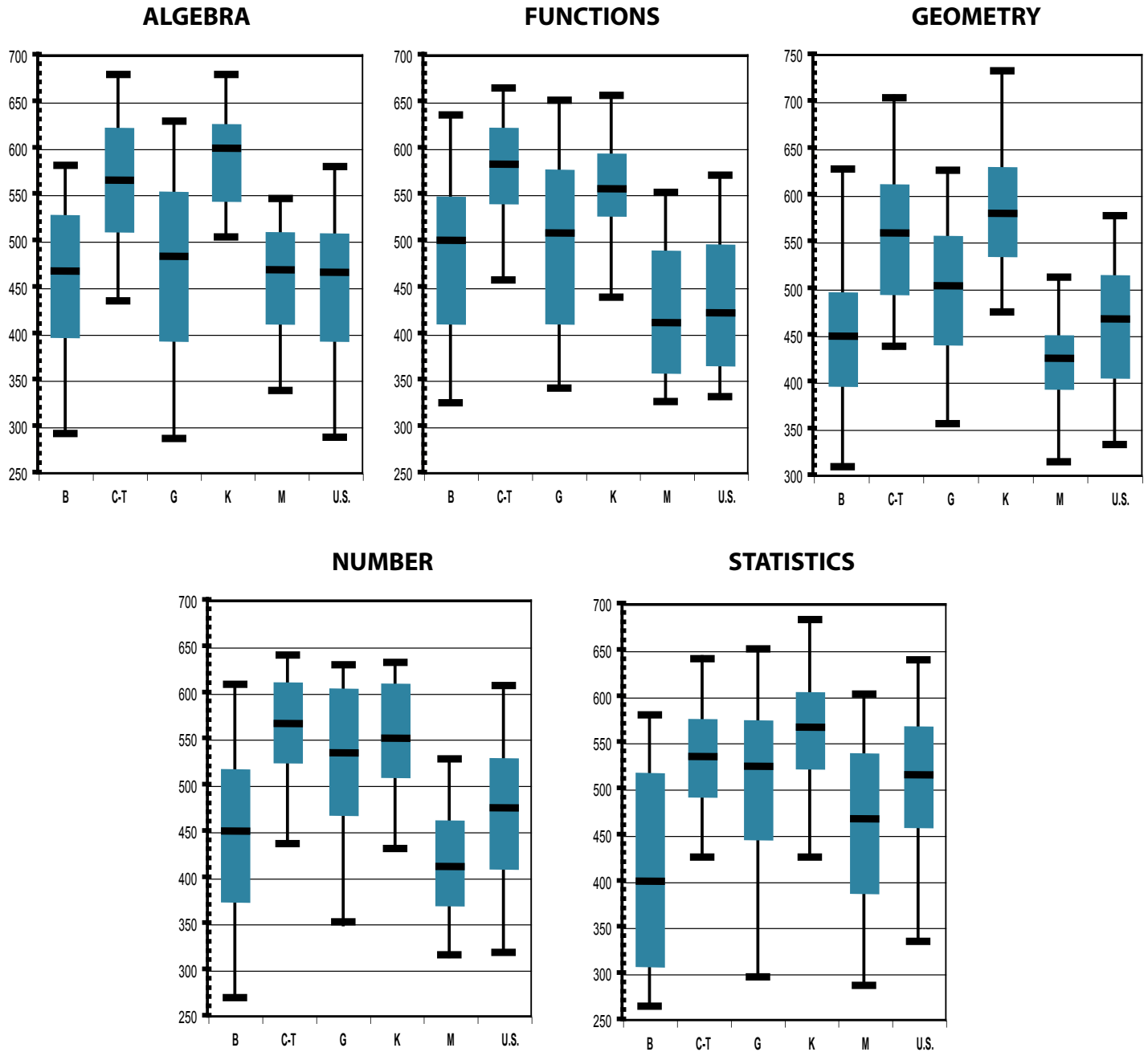
As the items were distributed over the two forms, the Rasch model (one-parameter Item Response Theory model) was used to obtain scale scores based on the item classifications. There were 15 dimensions – combinations of mathematics knowledge and mathematics pedagogy – available from the design blueprint for the tests. Each set of items pertained to one of the 15 dimensions. Each set was included in a separate Rasch scaling.

The scaled scores for the five mathematics knowledge areas and the three mathematics pedagogy areas were computed from these Rasch scores. Latent covariance structure analysis was used in which the pattern defining the relationships between the 15 dimensions and the eight scales was only partially specified. The remaining parameters were estimated by maximum likelihood procedures and used to derive estimates of the eight scales. The resulting eight scales were re-scaled so that each had an international mean of 500 and a variance of 100.

This approach takes advantage of the fact that all items measured knowledge in some area of mathematics and, additionally, some of the items also measured pedagogical knowledge. Those items contributed to knowledge both in pedagogy and in mathematics. The resulting pedagogical scales were conditioned on knowledge of the mathematics in which the pedagogical knowledge was situated. In that way, the pedagogical scales reflect pedagogical knowledge controlling for the level of knowledge associated with the mathematics itself. A more detailed description of

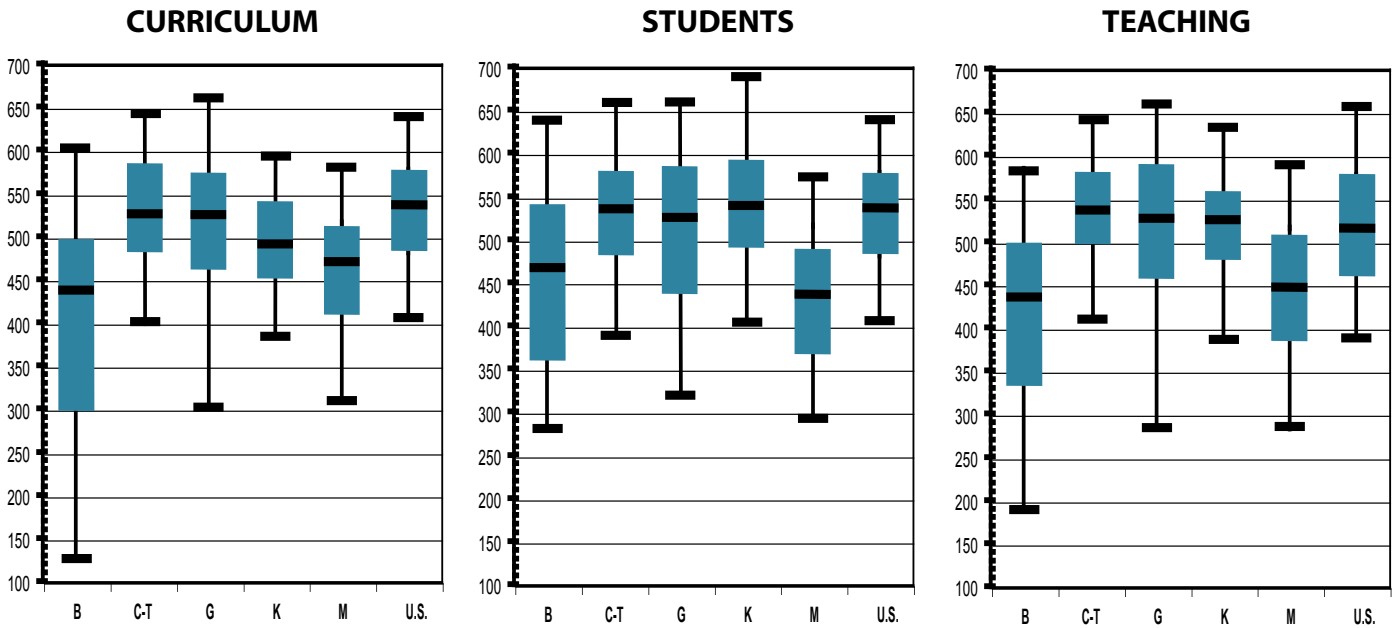
these methodological issues will be found in subsequent reports from this study. Figures 4.1 and 4.2 show, respectively, the six country percentile distributions for each of the five mathematics scales and for each of the three pedagogical knowledge scales. For each country on each scale there was substantial variation across the future teachers.

**Figure 4.1: Percentile Distribution of Mathematics Knowledge Scale Scores Across Students Within Each Country**



\*For this and all subsequent figures, the following abbreviations were used for the countries: B—Bulgaria, C-T—Chinese-Taipei (Taiwan), G—Germany, K—South Korea (Korea), M—Mexico, U.S.—United States

**Figure 4.2: Percentile Distribution of Mathematics Pedagogical Knowledge Scale Scores Across Students With Each Country**

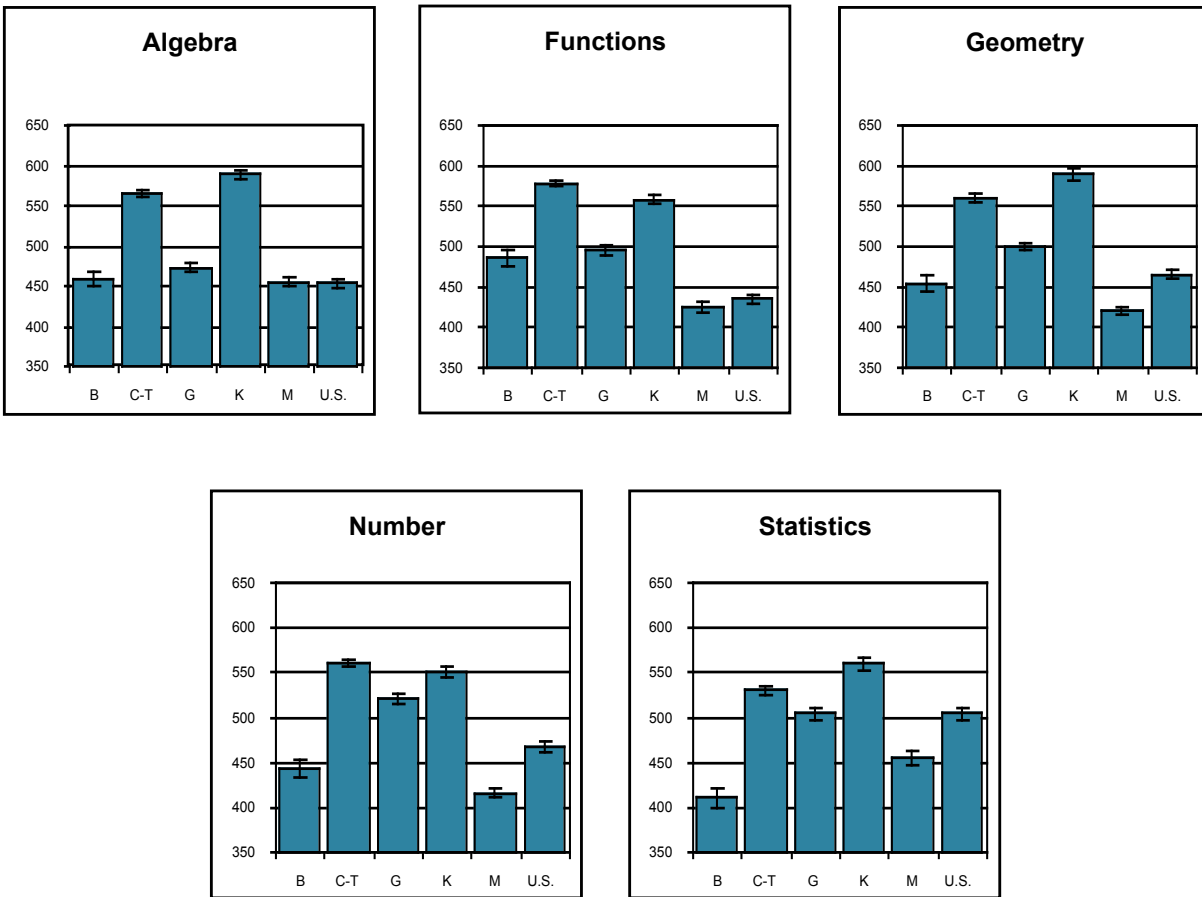


## ■ CROSS-NATIONAL RESULTS

**Mathematics Knowledge:** The differences across the countries on each of the five mathematics scales were statistically significant ( $p < .0001$ ).<sup>\*</sup> Figure 4.3 indicates that Korea and Taiwan each scored somewhere from around one-fourth to over a full standard deviation above the means of each of the other four countries on all five of the mathematics scales. Germany was typically in the middle of the international distribution while Mexico was well below the international means on most scales. Bulgaria and the US varied in their performance depending on the scale. In general, they scored anywhere from the middle of the distribution to almost one full standard deviation below the six-country mean.

<sup>\*</sup> The standard errors were computed taking into account the fixed clustering effect. The institutions were considered fixed as they were not randomly sampled. The unit of analysis used in this chapter was the individual teacher. The resulting estimated standard errors tend to be somewhat liberal.

**Figure 4.3: Mean Level Performance on the Mathematics Knowledge Scale Scores Across the Six Countries**



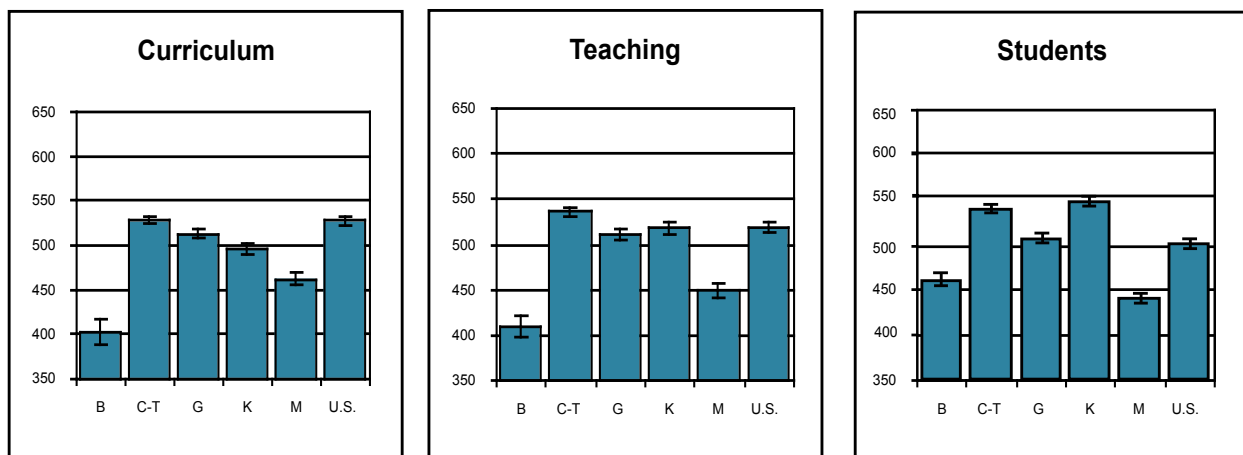
From the point of view of most countries studied around the world in TIMSS, the middle grades' curriculum centers on geometry and algebra. It is typically where students move from the more computational aspects of arithmetic to the more abstract notions of mathematics as a discipline. How well the future teachers in each of these countries performed on the algebra and functions scales is particularly important given how central algebra is to the middle school curriculum. The US future teachers' performance was at its worst in this area. The US future teachers' performance was essentially at the bottom of the international distribution in the area of algebra. The US future teachers' performance in the area of functions was the second lowest, surpassing only Mexico. The only bright spot in US future teachers' performance was in statistics. The US performance was around the six-country mean for statistics.

## Mathematics Pedagogy Knowledge

What did a mathematics pedagogy item look like? One example is an item that asked future teachers to determine the correctness of different answers given by hypothetical students to a common mathematics problem and to determine the nature of the error. This item depends on an understanding not only of the relevant mathematics involved but also of how students reason mathematically and what are their common misconceptions.

The three scale scores reflect knowledge of the lower-secondary curriculum, how students think and learn mathematics (hereafter, “student”) and instructional approaches for teaching particular mathematics topics (hereafter “teaching”). Figure 4.4 presents the results for these three scales. The differences among the countries were statistically significant ( $p < .0001$ ) for all three scales. However, most of the cross-country differences were smaller than those for the five mathematics scales. The rankings of the countries differed somewhat from those pertaining to mathematics knowledge. On the curriculum test, for example, Taiwan and the US performed the best. For the test focused on student reasoning, the rankings were more like those for mathematics knowledge with Taiwanese and Korean future teachers performing about a quarter to one standard deviation above the means of each of the other four countries. Germany and the US were in the middle of the six-country distribution.

**Figure 4.4: Mean Level Performance on the Mathematics Pedagogical Scale Scores Across the Six Countries**



## ■ COMPARISON OF THE THREE US PREPARATION PROGRAMS

Three different types of programs train US future teachers. They can be trained by an elementary program, which typically prepares teachers to teach anywhere from K-8. They can be prepared in a specific middle school program that focuses on sixth through eighth grades. They can be prepared in a program of secondary training in which future teachers are prepared to teach typically anywhere from sixth or seventh grade through twelfth grade.

An examination of the performance of the US future teachers trained under any of these three programs reveals some important differences in their mathematics knowledge. When contrasting the performance of future teachers trained in a secondary program to those trained either in an elementary or middle school program, one finds large and statistically significant differences across the five areas of mathematics ( $p < .0009$ ).

On the algebra, functions, geometry and number tests, the secondary trained future teachers outperformed their counterparts in the other two programs by around one-half to three-fourths of a standard deviation. On the statistics test, the difference in performance was slightly less than one-half of a standard deviation. Such large differences in performance could be due to the nature of the students that enter into the three different programs. It could be a result of their course experiences while in that program. It could be and most likely is some combination of the two.

The future teachers in MT21 were all designated by the university as future teachers of middle school mathematics. Even though the students were prepared in a particular program (for example, in an elementary program), the testing was only done for those students within that program whose focus was on teaching middle school mathematics. The same was true with the secondary program. In the next part of this report, we present data describing the curricular experiences of the students in each of the three types of different types of preparation programs.

The differences across the three preparation programs were not statistically significant for the curriculum and teaching pedagogical scales. For the student learning pedagogy scale, future teachers prepared by a secondary program scored one-third of a standard deviation higher ( $p < .0036$ ) than those future teachers prepared in either of the other two programs.

US future middle school mathematics teachers trained in a secondary program were slightly below the mean of the six countries on the tests for algebra, functions and geometry. For the number test, those trained in the secondary program were actually slightly above the international mean. On the statistics test, those future teachers scored about one third of a standard deviation above the mean – a result that tied them with Taiwan for the second highest score among the six countries.

\*To account for multiple contrasts, a procedure based on the Bonferoni Inequality was used to determine statistical significance. The criterion used was .0083.



## V. WHAT ARE THE COURSE TAKING EXPERIENCES OF FUTURE TEACHERS IN EACH OF THE SIX COUNTRIES?

One possible explanation for the cross-country differences in achievement seen in Chapter IV of this report is that they are related to differences in the course taking or content opportunities associated with the teacher preparation programs in each of the six countries. We summarize the reported educational experiences of the future teachers in this section.

We examined this using two kinds of questions from the future teacher surveys. First, future teachers were asked whether they studied certain mathematics topics in one or more of the courses they took as a part of their program of study. The choices included the topics typically studied in a mathematics undergraduate program.

The second set of questions asked future teachers to rate the extent to which they had had the opportunity in their teacher preparation program to study various topics or to be engaged in specific activities. The six-point scale went from “not at all” (0) to “a great extent” (5).

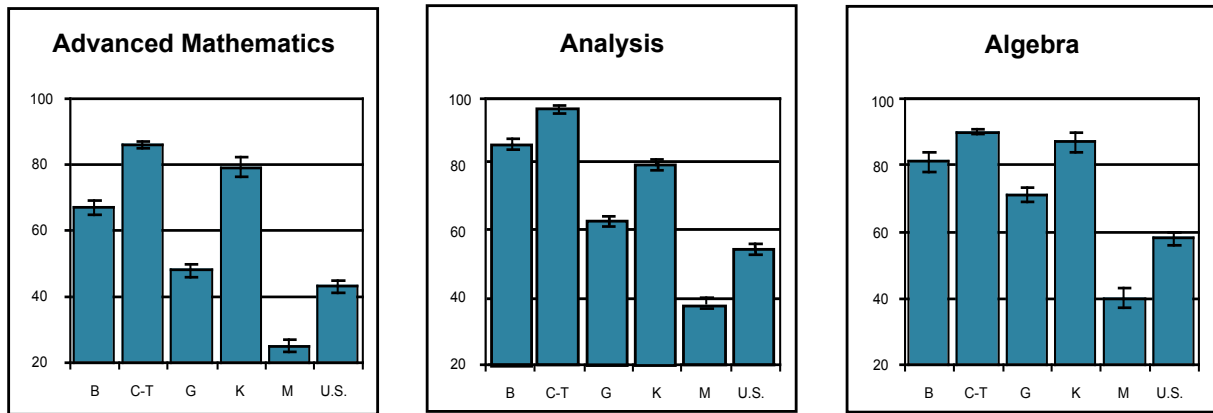
### ■ COURSE TAKING IN ADVANCED MATHEMATICS

The differences among the future teachers in the six countries as portrayed in Figure 5.1 were substantial and statistically significant ( $p < .0001$ ). For the future teachers in the two Asian countries, Taiwan and Korea, the proportion of the advanced topics studied by the typical future teacher was between 79 and 86 percent. Advanced mathematics included the topics of abstract algebra, calculus, multivariable calculus, differential equations, functional analysis (including the theory of real functions), the theory of complex functions, differential geometry and topology.

Bulgarian future teachers were similar with an average of almost 70 percent. For Germany this percentage dropped to around half of the topics being typically covered while future teachers in the U.S and Mexico typically covered less than half of the advanced topics. In fact, the typical future teacher in Mexico studied only around a quarter of the advanced topics in his or her teacher preparation program.

As indicated in Chapter III, in Germany and the United States there is more than one route or program towards certification to teach in middle school grades. As a result, the means as presented throughout this report for these two countries reflect the weighted average across all types of programs leading to certification. The resulting mean is meaningful in the sense that if all of these future teachers enter the work force, this statistic represents the typical mathematics preparation associated with what a student within that country would encounter, at least for the students of first year teachers. However, some of the middle school students could encounter

**Figure 5.1: Mean Percent of the Topics Covered in the Preparation Program in Each Country**



teachers with a significantly higher level of preparation in advanced mathematics if there were differences in preparation among the programs.

The differences between the three types of teacher preparation programs in the US were statistically significant ( $p < .0001$ ). For those future teachers prepared through the secondary program, the typical percentage of advanced topics covered was around 20 percent more than was the case for teachers prepared through the elementary or middle school programs ( $p < .0001$ ). There were no differences in preparation between the elementary and middle school programs ( $p < .47$ ).

## ■ COURSE TAKING IN ANALYSIS AND ALGEBRA

The cross-country differences for the topic of analysis (as found in Figure 5.1) were similar to those described for the amount of advanced mathematics taken by a typical future teacher in each country. However, there were some important differences. The difference between Korea and Taiwan, which was minimal for advanced mathematics, was somewhat larger for topics related to analysis, which includes beginning calculus, calculus, multivariable calculus, differential equations, analysis and the theory of complex functions.

The Taiwanese future teachers typically studied virtually all (96 percent) of the analysis topics. Korean future teachers studied about three-fourths (79 percent) of the topics on average. Bulgaria was between Korea and Taiwan in the typical coverage of topics related to functional analysis. Germany was in the middle of the six-country distribution. The US and Mexico again lagged behind in covering analysis topics.

The difference between the future teachers prepared in a secondary program compared to those prepared in an elementary or middle school program was similar to that for advanced mathematics with a difference in topic coverage of 22 percent ( $p < .0001$ ). This is a particularly

significant difference because school algebra (which includes both equations and functions) plays such an important role in the middle school curriculum, especially at eighth grade. The difference in background involving analysis topics suggests that secondary-trained middle school teachers would have a much more extensive background.

Data from a study of sixty districts in Ohio and Michigan (PROM/SE, 2006) indicated that only around one third of in-service middle school mathematics teachers had a major or minor in mathematics. Secondary- teachers (those with the more extensive background in functions) will not teach most children in US middle schools if these data are at all representative.

For algebra (see Figure 5.1), the differences among the countries were also statistically significantly ( $p < .0001$ ) as were the differences between the differing programs within the United States. The gap between the secondary program and the other two was about 35 percent ( $p < .0001$ ). This index was based on two topics — abstract algebra and linear algebra.

## ■ COURSE TAKING IN GENERAL PEDAGOGY

General education or pedagogy, as used in this report, refers to the practical aspects of teaching not in a particular subject matter but more generally. It does not refer to the more formal general education courses such as educational psychology, sociology of education or child development. The extent of the opportunities, as reported by the future teachers, was measured on a six-point scale ranging from zero (“not at all”) to five (“to a great extent”).

MT21 used an interpretive framework that a mean value greater than three indicates that the opportunities provided in the teacher-training program were adequate. The results presented in Figure 5.2 show that only Mexican and Taiwanese future teachers typically believed in the adequacy of the opportunities provided. The three items used to define general pedagogy included learning how to “incorporate classroom management strategies into teaching,” “use student data to improve teaching” and “contribute to school development.”

For Bulgaria, the US and Korea, future teachers indicated a more modest amount of opportunity was provided (see Figure 5.2). By contrast, German future teachers indicated they had been provided little or no coverage in the practical aspects of general pedagogy. This is true comparatively since the mean for the German future teachers was about two points lower than that for Taiwan and about one point lower than that for the US. It is also true absolutely as a mean value of 1.28 would indicate. The magnitude of the differences across the institutions studied in these six countries is statistically significant ( $p < .0001$ ), large and likely reflects truly substantial differences in teacher preparation, at least as reported by the future teachers.

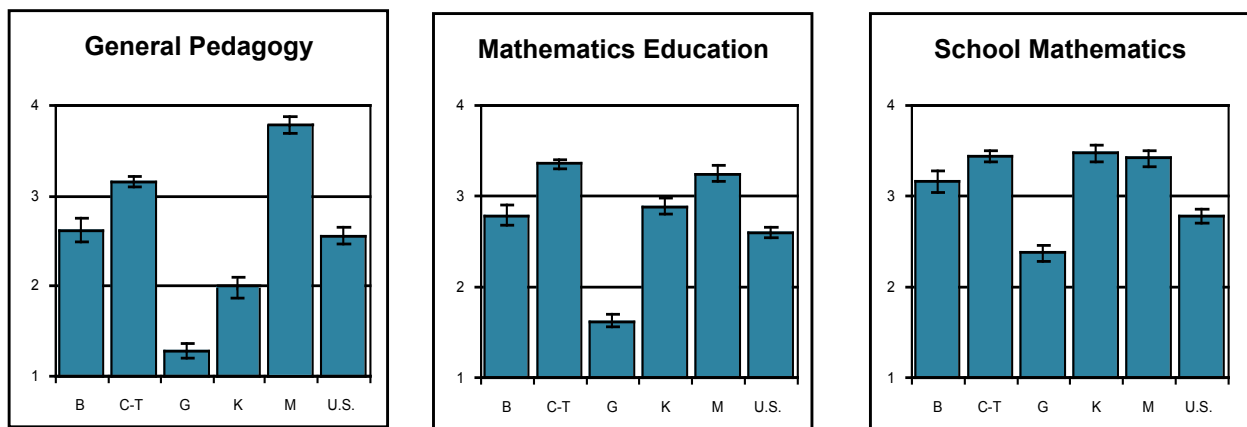
In the US, no significant differences ( $p < .077$ ) were found in preparation in general pedagogy between those future teachers of middle school mathematics prepared in an elementary program compared to those prepared by either a middle school or secondary program. Although not significant, the future teachers prepared through an elementary program indicated that the extent of coverage was greater than was the case for the other two programs.

## ■ COURSE TAKING IN MATHEMATICS EDUCATION

Similar results, as illustrated in Figure 5.2, were obtained for the opportunities related to the practical aspects of teaching mathematics. The future teachers in Mexico and Taiwan again had the highest means, indicating adequate opportunities, while those in Germany the lowest, again indicating little opportunity. Korean, Bulgarian and US future teachers indicated coverage that was more modest.

Coverage in mathematics education was defined as the average response across seven items each dealing with some aspect involved in providing instruction in mathematics to middle school students. These items were not about the theory or history of mathematics pedagogy. They dealt more with the practical aspects of actually teaching mathematics. The items from the questionnaire that define this dimension include such things as learning how to: “teach students how to use algorithms,” “teach students how to do mathematical proofs” and “present middle school mathematics in ways that build on students existing understandings.”

**Figure 5.2: Mean Rating of the Extent of Coverage of Practical Pedagogy in Each Country**



Another aspect of mathematics education focuses on the difference between mathematics as a research discipline and mathematics as it relates to schooling. It is about mathematics education but is more about mathematics than pedagogy as usually defined. It is about the mathematics that has a direct relationship to the teaching of school mathematics (Hill, 2007; Hill, Ball & Schilling, 2004; Hill, Rowan & Ball, 2005).

Cross-country differences on the opportunity provided to future teachers for study of the mathematics underlying school mathematics were statistically significant ( $p < 0.0001$ ). Coverage of the mathematics related to school mathematics was based on five items that ask the extent to which future teachers had had opportunities to gain a deeper understanding of the five topics

included in the typical middle school curriculum — number, geometry, algebra, functions and statistics.

This dimension is about opportunity to study mathematics but differs from the number of advanced mathematics topics covered. It is about the advanced treatment of topics that are traditionally part of the middle school curriculum. These are not topics that mathematics students not interested in teaching would typically study.

The mean values for future teachers from Taiwan, Korea, Mexico and Bulgaria were all greater than three, which suggests satisfaction in the extent of the opportunities provided.

The two countries where this was not the case were Germany and the US. The value of the mean for Germany was higher than was the case for the two previous pedagogical dimensions but was still the lowest among the six countries. There were no statistically significant differences among the three US programs on this.

## ■ SUMMARY

The results described in the previous sections indicate that US future teachers did not feel that they had had adequate opportunities to master the more practical aspects of teaching in general and, more specifically, of teaching mathematics. In both cases the means were below three and were significantly lower than that reported by Taiwanese future teachers. This does not mean that US future teachers did not take general education or mathematics education courses. We know from other data that they did. Rather, this indicates that, in their judgment, even before they entered the classroom they did not feel those experiences had given them adequate opportunities to learn important practical aspects of their future job. Elementary programs did more of this, but as indicated in a previous section, they covered less mathematics.

One issue that begins to emerge from these data concerns the preparation of the middle school teachers through programs aimed specifically at the middle school. It would appear that in some ways the training associated with that program provides teachers with the worst of both worlds. Their opportunities in mathematics were not different from the opportunities provided by the elementary program but were substantially different from the preparation provided by the secondary program — a difference on the order of covering 20 or 35 percent fewer topics. On the other hand, those same future teachers received less practical pedagogical training than was the case in the elementary program. In this case, they are trained more like those teachers prepared in a secondary program.

Using an international benchmark, defined by the preparation of future teachers in Taiwan and Korea (countries that have high performing students at the eighth grade) the US future teachers studied many fewer topics in advanced mathematics and indicated they had somewhat less opportunity in terms of the amount of practical pedagogy studied. More specifically in mathematics, the coverage of advanced mathematics topics was only about half of what is

typically taught in undergraduate mathematics programs. In Taiwan, the typical future teacher covered around 90 percent of those topics. In pedagogy, they received a modest amount of coverage compared to Taiwan's strong and, in some cases, extensive coverage. Those prepared by the secondary program had fifty to eighty percent coverage of advanced mathematics topics and a modest amount of pedagogy. This amount of preparation is certainly more similar to but not equivalent to the international standard.

Differences across countries in the opportunities that future teachers had in their teacher preparation program together with the concomitant differences on the knowledge scales suggest the likelihood that teacher education matters. In addition, supporting what at this point is only a conjecture, the countries varied in their performance depending on the particular area of mathematics assessed. For example Germany was about one-fourth of a standard deviation above the mean of the six countries in the area of number but in most of the other areas was essentially at the mean and in the case of algebra below the international mean.

Those differences among the countries in terms of mathematics achievement combined with the differences between the programs both within Germany and the US would in general support the hypothesis of a curriculum or teacher education effect. When contrasting the two German program types, the one area in which differences between the two German programs were relatively small was that of statistics. Like the United States, Germany has focused on statistics as an integral part of the K-8 curriculum.

Teasing out the nature of the relationship, while controlling for other factors, awaits further, more sophisticated, analysis. We point to the variation across countries, and across programs within Germany and the US, only as an indication. It perhaps indicates that what future teachers are required or choose to take as a part of their teacher preparation may well have an impact on how much knowledge they have, at least as measured in the five domains of mathematics tested in this study.

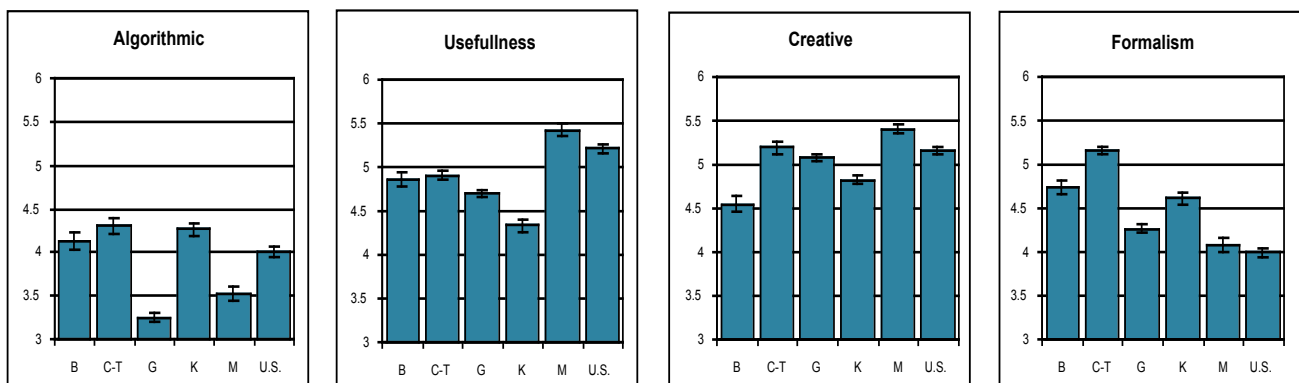
## VI. BELIEFS ABOUT MATHEMATICS

Future teachers have a number of beliefs. These include beliefs about the nature of the discipline of mathematics, what should be done in teaching mathematics, how mathematics is best learned by the students and who it is within the classroom that is most likely to succeed in the learning of typical middle school mathematics. These beliefs could influence what future teachers will do as they enter the classroom.

### ■ THE NATURE OF MATHEMATICS

Mathematics can be viewed as being formal, algorithmic, creative and useful. Future teachers indicated their degrees of agreement with items on a questionnaire related to the ideas about mathematics ranging from strongly agree to strongly disagree with four degrees of agreement or disagreement between the two extremes. The first thing that is noticeable from Figure 6.1 is that for all countries the average values on all of the scales, except “algorithmic,” were essentially four or higher on the six-point Likert scale indicating general agreement. There were a few notable exceptions to this pattern on the algorithmic scale. Even there, none of the mean values were three or lower. Hence, there was in general no disagreement at the country level with any of the four characterizations as to the nature of mathematics. There was, however, some degree of ambiguity on the algorithmic scale.

**Figure 6.1: Mean Rating of the Degree of Belief About the Nature of Mathematics Across the Six Countries**



Items defining “algorithmic” include, among others, “mathematics is a collection of rules and procedures that prescribe how to solve a problem” and “mathematics involves the remembering and application of definitions, formulas, mathematical facts and procedures.” Mathematics can also be viewed as a “formal” discipline that is defined by such items as “mathematical thought is characterized by abstraction and logic” and “hallmarks of mathematics are clarity, precision and unambiguousness.” Mathematics is also characterized as “creative.” The items included

“mathematics means creativity and new ideas” as well as “if one engages in mathematical tasks, one can discover new things (e.g. connections, rules and concepts).” The items that define “usefulness” include such things as “mathematics is useful for every profession” and “many aspects of mathematics have practical relevance.”

Several differences are noteworthy across the countries and across the four scales (the cross-country differences are statistically significant for all four scales [ $p < .0001$ ]). In Mexico, for example, the typical future teacher was somewhat neutral on the belief that mathematics is algorithmic in nature, but indicated strong agreement with its potential usefulness in applications as well as the view that mathematics is creative. In Taiwan, Korea and Bulgaria, there was general agreement across all four scales indicating that the future teachers in those countries tended to believe that mathematics is characterized by all four descriptions. It is notable that the typical Taiwanese future teacher indicated one of the strongest degrees of agreement among the countries on each of the four scales but especially so for recognizing the more formal disciplinary nature of mathematics.

The German future teachers had the lowest mean for the index describing mathematics as being algorithmic. If rounded to the nearest whole number, their mean rating would indicate slight disagreement with such a characterization.

The main difference among the US programs of teacher preparation was that future mathematics teachers prepared in elementary or middle school programs more strongly agreed with the description of mathematics as being algorithmic than did the future teachers prepared through secondary programs ( $p < .0003$ ). Those future teachers prepared in the middle school program agreed more strongly than did those prepared in the elementary program ( $p < .0004$ ).

## ■ OBJECTIVES FOR MATHEMATICS INSTRUCTION

This dimension measures the perception of future teachers of emphasis they would give to a series of student objectives that could be considered in the teaching of middle school mathematics. The scale for an individual item ranges from zero to five where zero indicates “no emphasis” should be given to the objective while five indicates that “a great deal of emphasis” should be placed on the objective. For purposes of discussion, we dichotomized the index taking a mean value of four or higher as indicative that strong emphasis should be given to the objective.

The questionnaire listed 28 different objectives that were grouped into six scales:

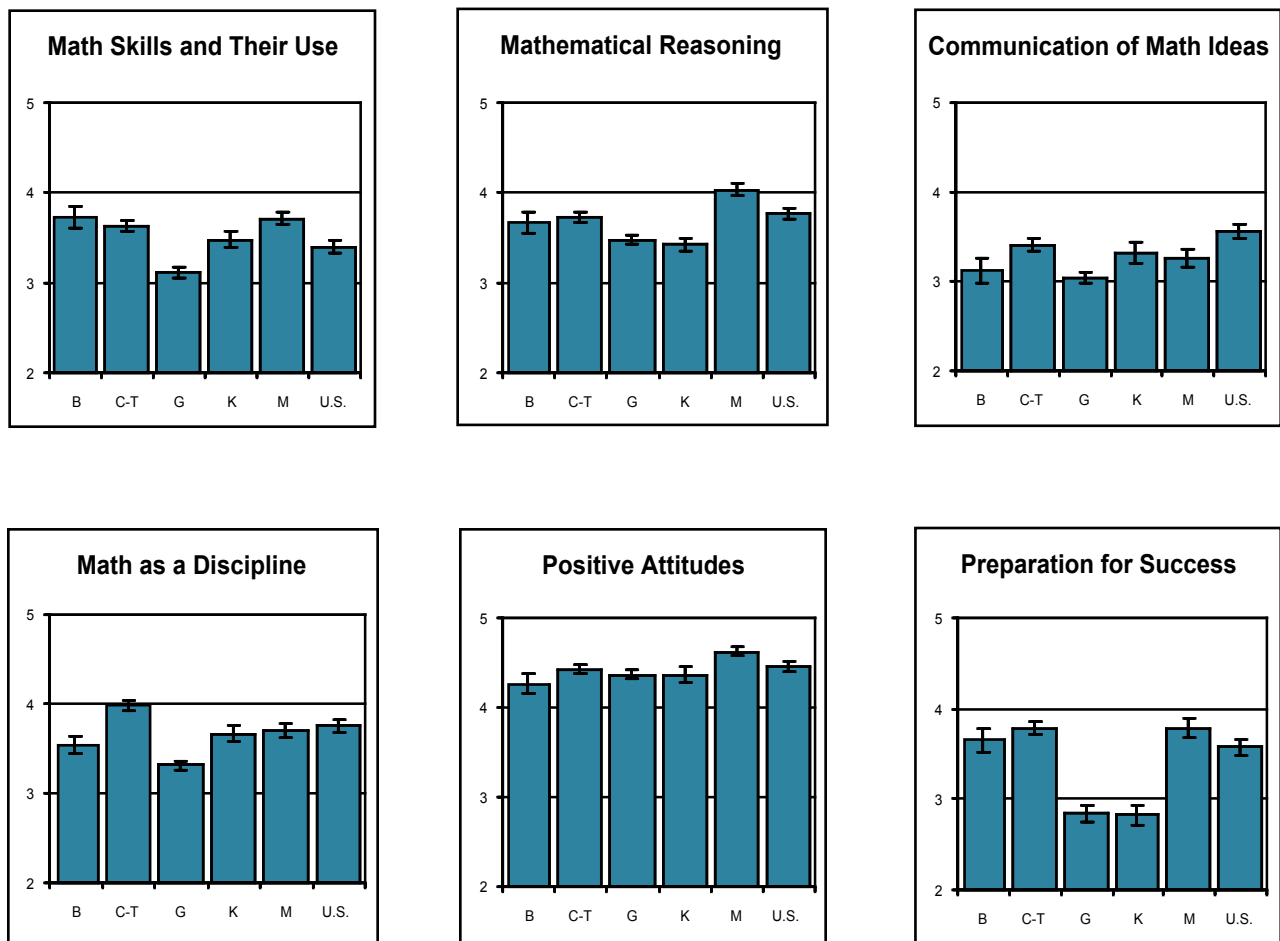
- (1) Mathematics skills and their use;
- (2) Mathematical reasoning;
- (3) The ability to communicate mathematics ideas;
- (4) Student understanding of mathematics as a formal discipline;
- (5) Positive attitudes towards mathematics; and
- (6) Better preparation to succeed in school and in future mathematics courses.



For mathematics skills, mathematics reasoning and mathematics as a formal discipline, the means (see Figure 6.2) suggest that across all countries future teachers indicated moderate to strong emphasis should be given to these objectives. Mexico was notable — it had one of the highest mean values on both skills and reasoning. Germany was also notable, having the lowest mean on both mathematics skills and mathematics as a formal discipline (both below 3.5).

Mathematics skills included, among others, such items as “knowing mathematical algorithms,” “memorizing facts, rules and steps” and “developing an awareness of mathematical uses in everyday life.” Objectives focusing on the student’s ability to think mathematically — that is,

**Figure 6.2: Mean Belief In the Extent of Emphasis That Should be Given to Various Objectives for the Six Countries**



to be able to do mathematical reasoning — included such items as “learn to solve problems not only with procedures and techniques learned in school but also those that require creativity and imagination in order to be solved” as well as “reason mathematically” and “to model real world problems.” The formal structure of mathematics as a discipline included items such as

“understand the logical structure of mathematics” and “learn how mathematical ideas connect with one another.”

The objectives related to better future preparation were not strongly emphasized by Korean or German future teachers. In fact, the mean values for the future teachers in those two countries were among the lowest for any of the subscales. The scale includes objectives such as “being prepared for further studies in mathematics” and “being prepared for high school exit exams or university entrance exams.”

Similarly, the mean values for objectives related to communication tended to be lower than for the other subscales, not just for Korea and Germany but also for all countries. The mean value for Germany was once again the lowest among the six countries. Objectives which focus on the ability of students to communicate mathematical ideas included items such as “the ability of students to learn to write mathematical essays” and “to be able in general to communicate about mathematics.”

For objectives related to developing positive attitudes towards mathematics, future teachers in all six countries left their teacher preparation programs clearly intending to allocate extensive emphasis to affective objectives. The scale for this included, among other items, “having the student feel that mathematics is something he or she can do” or “to have students become interested in mathematics.”

There were no differences among US future teachers across the three programs of preparation for any of the scales related to the different types of objectives.

## ■ HOW STUDENTS LEARN MATHEMATICS

Beliefs about how students learn mathematics might predict the particular instructional strategies that these teachers would be inclined to use, at least as they leave their teacher preparation program but before they actually begin to teach mathematics in schools. Five indices were developed based on the 18 items to which the future teachers indicated their degree of agreement.

The scales indicate the degree of agreement as to whether students learn by:

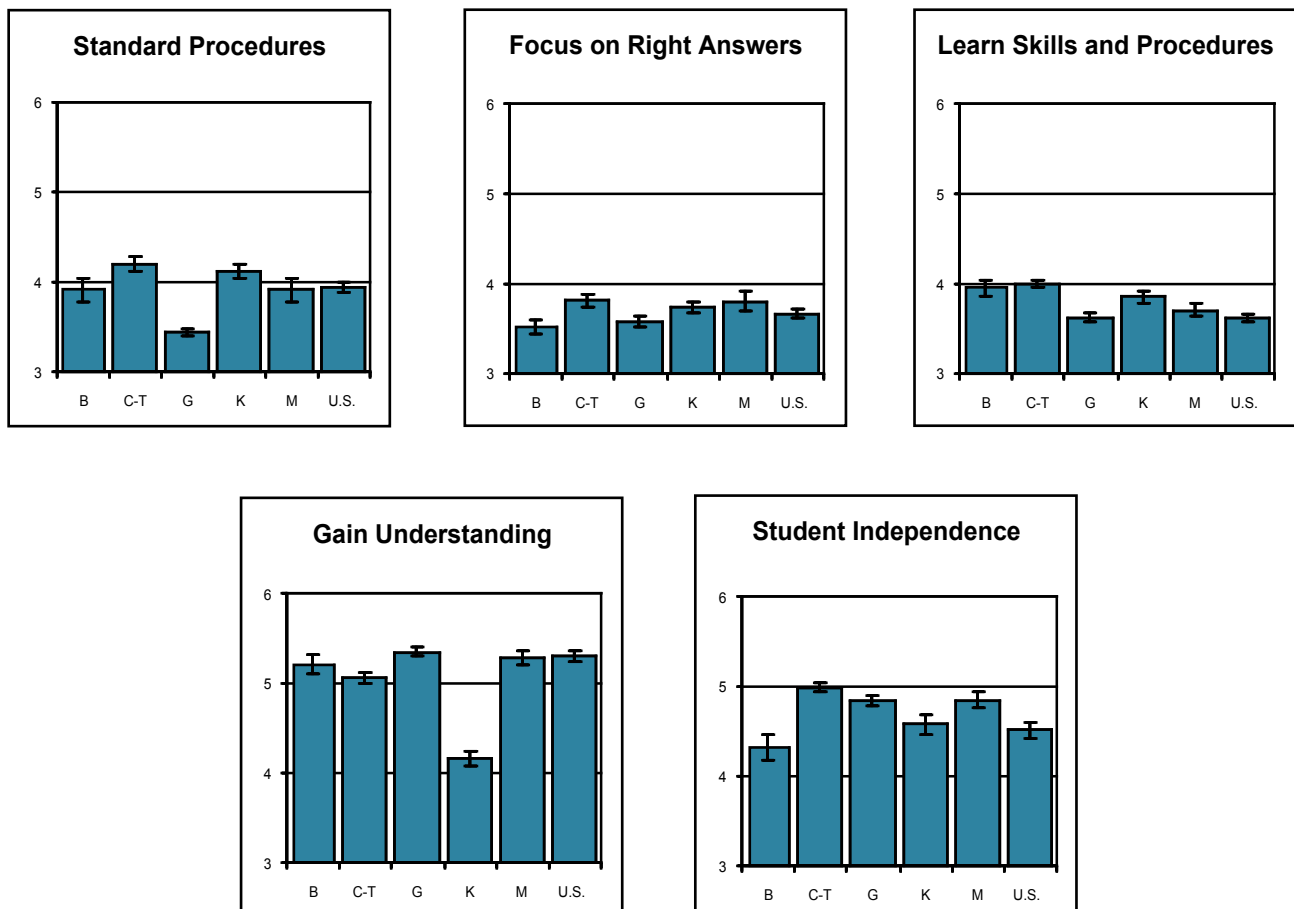
- (1) The use of standard procedures,
- (2) Focusing on the right answer more than the procedure,
- (3) Learning and mastering skills and procedures,
- (4) Gaining understanding,
- (5) Focusing on students being independent in their work.

The country means varied between 3.4 and 4.2 for the first three scales (see Figure 6.3) indicating no disagreement in any of the six countries (although a mean of 3.4 indicates a somewhat neutral attitude but not disagreement). The differences among countries on getting the right answer were

not statistically significant ( $p < .08$ ) but they were on the other two scales ( $p < .0001$ ). Future teachers in Taiwan had the highest mean on the use of standard procedures.

Use of standard procedures is actually more the avoidance of non-standard procedures. Two items defined this belief: “nonstandard procedures should be avoided” and “hands on mathematics experiences aren’t worth the time and expense.” Belief that students learn mathematics by focusing on getting the right answers more than on the procedures includes items such as “it doesn’t really matter if you understand a mathematical problem if you can get the right answer” or “when students attempt to solve problems, more emphasis should be put on getting the correct answer than on the process followed.” Learning and mastering skills and procedures were defined by items such as “for students to get better at mathematics they need to practice a lot” and “the best way to do well in mathematics is to memorize all the formulas.”

**Figure 6.3: Mean Rating of the Degree of Belief in Different Ways That Students Learn Mathematics Across the Six Countries**



Belief that gaining understanding is the way students learn mathematics received the highest endorsement on the part of the future teachers in five of the countries. The only exception was Korea where the mean of four, although high, was lower than the means of the other five countries (which were all greater than five). The US, Mexican, and German means were the highest among all the countries ( $p < .0001$ ).

Helping the students to gain an understanding of mathematics includes such items as “in addition to getting the right answer in mathematics, it is important to understand why the right answer is correct,” “time used to investigate why a solution works is time well spent” and “it is helpful for students to discuss different ways to solve different problems.”

The extent to which future teachers carry this belief into the classroom could have a major impact on instructional practice and especially on time allocation. In order to implement such a practice effectively, the teachers would need a deep understanding of the underlying mathematics. Given the cross-country variability in mathematics achievement on the part of the future teachers, implementing such practices could have different consequences in different countries. This could, for example, be an issue in the US.

The typical future teachers from all six countries agreed that students learn by working independently – a belief likely implying a particular type of instructional approach. All the means in this case ranged between four and five. Typical items in this scale include “teachers should allow students to find their own ways to solve mathematics problems” and “students can figure out a way to solve problems without a teacher’s help.”

What about future teachers prepared by the three different types of programs? There were no statistically significant differences in their beliefs on how students learn.

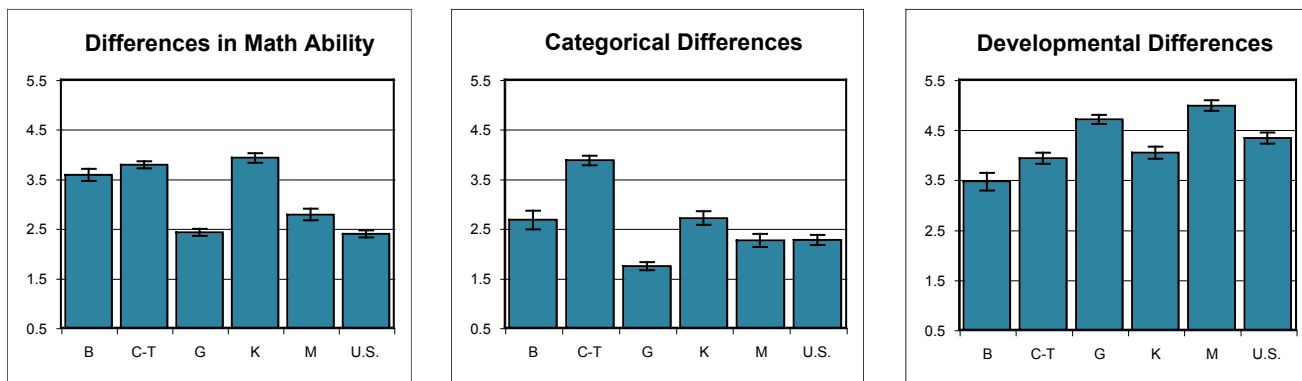
## ■ INDIVIDUAL DIFFERENCES IN MATHEMATICS

The last dimension focuses on future teachers’ beliefs regarding why children perform so differently on mathematics tests. In other words, what beliefs do they hold about why some children do particularly well in mathematics while others struggle to achieve even the barest minimum accomplishment in mathematics. Three indices were created based on 10 Likert type items. The first scale reflected the belief on the part of the future teachers that differences in mathematics achievement reflect differences in natural ability. The second scale focuses on categorical differences such as ethnicity and gender. Agreement with the last scale indicates a belief that there simply are developmental differences among students on this dimension as there are on other traits.

Consider first the belief that differences among children in mathematics achievement are mostly related to differences in natural ability. The means across the six countries were all less than four, as Figure 6.4 shows, indicating some level of ambiguity or disagreement. However, for Bulgaria,

and especially Korea and Taiwan, where the means were close to four, there was a degree of belief, although not particularly strong, that natural ability is one of the primary explanations of individual differences in mathematics achievement. However, clearly for the German, Mexican and US future teachers, there was mostly disagreement with this position ( $p < .0001$ ).

**Figure 6.4: Mean Degree of Belief That Variation in Mathematics Achievement is Related to Various Factors for the Six Countries**



In the case of categorical differences one finds somewhat of a different pattern. The differences among the countries were statistically significant ( $p < 0.0001$ ) and indicated that Taiwan again had a mean value similar to that on the scale of natural ability. For the typical German future teacher, this notion was rejected as an explanation for variability in mathematics achievement.

For the Bulgarian future teachers there was not strong agreement with any of the three explanations of individual differences although the one that received the last positive response was the one focused on categorical differences. In Korea, natural ability and developmental differences were the predominate explanations for individual differences.

For the US, Mexican and German future teachers, the issue was clear. They did not endorse either categorical or natural ability as related to variability in mathematics achievement but believed strongly that such variability was related to developmental differences. The Taiwanese future teachers on the other hand believed moderately that all three are related to variability in achievement.

There were statistically significant differences among future teachers prepared by the different programs in the US in their beliefs on this scale. Future teachers prepared in the middle school program believed more strongly that natural ability accounted for differences among students in terms of their learning mathematics than did future teachers prepared through an elementary program ( $p < .0026$ ). No differences were found across programs regarding beliefs about categorical or developmental differences.

## VII. WHAT ARE THE POLICY IMPLICATIONS?

It is tempting, based on the results from the six countries participating in Mathematics Teaching in the 21<sup>st</sup> Century (MT21), to make recommendations to change the nature of US middle school teacher preparation in mathematics. The results clearly suggest that teacher education as defined by the learning opportunities provided likely has an impact on what future teachers know and believe as they leave their teacher preparation program. The differences across the countries, combined with the differences across the three programs of preparation within the United States, all point in that direction.

Those countries whose students have performed well in previous international comparative studies demanded a different level of preparation on the part of their future teachers than was provided in the United States. In Taiwan and Korea, the level of mathematics preparation was very strong and, in both countries, the amount of emphasis given to the practical issues of mathematics pedagogy was also extensive. In general pedagogy, there was a difference between the two top performing countries. In Taiwan, there was also extensive emphasis given to the practical aspects of general pedagogy such as classroom management, which was not the case for Korean future teachers. Apart from that difference, it is clear that the future teachers in those two countries have extensive coverage of two of the areas (and in the case of Taiwan all three areas). In none of the six countries was the preparation of teachers done without at least some level of coverage associated with each of the three broad areas.

The profile of coverage associated with the preparation of future teachers in Germany was quite different than that of the Asian countries. The preparation in mathematics was not as extensive as it was in either Taiwan or Korea but was still relatively high. German future teachers on average were expected to cover about 60 to 70 percent of the topics in algebra and analysis but essentially provided little if any emphasis to the coverage of topics associated with the practical aspects of general and mathematical pedagogy.

The contrast of the German profile to that of the future teachers in the US is interesting. The amount of mathematics taken by the future teachers in the US was lower in algebra and analysis than was the case for German future teachers, but the amount of opportunity that was provided in the pedagogy areas was much larger. On the international TIMSS test, eighth graders in both Germany and US had very comparable low levels of performance.

Although tempting, making such recommendations would be premature. The data are informative. We believe they are very likely representative of the preparation of future teachers in each of the six countries that participated in MT21. However, the samples were not drawn randomly and the extent to which they are representative rests only with the judgment of the scholars involved in this research. For that reason we simply present these results to begin a dialogue on what the implications of this work might be.

The good news is that a more extensive study is currently underway in which a nationally representative sample of US teacher preparation institutions was drawn and a related version of the MT21 study will be done within the United States and across some 20 countries (Teacher Education Study in Mathematics – IEA/TEDS-M). The focus of this report is only on middle school teachers. TEDS-M, not only will further study middle school teacher preparation but will also do the same for the preparation of elementary school teachers. That latter part of the study is totally new and will provide insights into the preparation of teachers for the lower grades.

We await the results of that study to draw specific recommendations for reforming teacher preparation in the United States. However, one implication of these results seems clear. Not only do we have a curriculum gap as identified in TIMSS for the students especially those in the middle grades, but we also have the likelihood of another gap — which we have named the preparation gap. The preparation gap indicates substantial differences associated with the preparation of future teachers in this country when contrasted with that of the other MT21 countries, especially the high performing ones.

We do recommend that discussions should be begun having to do with the amount of mathematics required in the training of middle school teachers. However, just as importantly, the belief that the preparation of future teachers might be done without any preparation in practical pedagogy seems unwise and should certainly be reconsidered. The fact that none of the five countries prepare their teachers in this way tells us something. The real question then is not whether such experiences are necessary but rather the nature and the extent of the learning opportunities in each of the three areas that should be available for future teachers. It is quite revealing that the countries whose students continuously perform well on the international benchmark tests have the teachers who have been trained with extensive educational opportunities in mathematics as well as in the practical aspects of teaching mathematics to students in the middle grades.

## REFERENCES

- Adler, J., Ball, D.L., Rainer, K., Lin, F.-L., & Novotna, J. (2005). Reflections on an emerging field: Researching mathematics teacher education. *Educational Studies in Mathematics*, 60, 359–381.
- Ball, D. L. & Bass, H. (2003). Toward a practice-based theory of mathematical knowledge for teaching. In Davis, B. & Simmt, E. (Eds.): *Proceedings of the 2002 annual meeting of the Canadian Mathematics Education Study Group*. Edmonton: CMESG/GCEDM, pp. 3-14.
- Baumert, J. & Kunter, M. (2006). Stichwort: Professionelle Kompetenz von Lehrkräften. *Zeitschrift für Erziehungswissenschaft*, 9, 469–520.
- Bishop, A.J. (1991). *Mathematical Enculturation. A Cultural Perspective on Mathematics Education*. Dordrecht: Kluwer Academic Publishers.
- Blömeke, S., Felbrich, A. & Müller, Ch. (in press). Theoretischer Rahmen und Untersuchungsdesign von P-TEDS. In Blömeke, S., Kaiser, G. & Lehmann, R. (Eds.), *Professionelle Kompetenz angehender Lehrerinnen und Lehrer. P-TEDS: Erste Ergebnisse zum Wissen, zu den Überzeugungen und zu Lerngelegenheiten deutscher Mathematik-Studierender und -Referendare im internationalen Vergleich*. Münster: Waxmann.
- Blömeke, S. (2002). *Universität und Lehrerausbildung*. Bad Heilbrunn: Klinkhardt.
- Blömeke, S. (2004). Empirische Befunde zur Wirksamkeit der Lehrerbildung. In Blömeke, S., Reinhold, P., Tulodziecki, G. & Wildt, J. (Eds.), *Handbuch Lehrerbildung* (pp.59–91). Bad Heilbrunn/Braunschweig: Klinkhardt/Westermann.
- Blömeke, S. (2005). *Lehrerausbildung – Lehrerhandeln – Schülerleistungen: Perspektiven nationaler und internationaler empirischer Bildungsforschung*. Antrittsvorlesung v. 10. Dezember 2003. Berlin: Humboldt-Universität (= Öffentliche Vorlesungen; 139).
- Blum, W., Neubrand, M., Ehmke, T., Senkbeil, M., Jordan, A., Ulfig, F. & Carstensen, C.H. (2004). Mathematische Kompetenz. In M. Prenzel, J. Baumert, W. Blum, R. Lehmann, D. Leutner, M. Neubrand et al. (Eds.), *PISA 2003. Der Bildungsstand der Jugendlichen in Deutschland – Ergebnisse des zweiten internationalen Vergleichs* (pp. 47–92). Münster: Waxmann.
- Bromme, R. (1992). *Der Lehrer als Experte. Zur Psychologie des professionellen Lehrerwissens*. Göttingen: Hans Huber.
- Bromme, R. (1997). Kompetenzen, Funktionen und unterrichtliches Handeln des Lehrers. In F.E. Weinert (Eds.), *Psychologie des Unterrichts und der Schule* (pp. 177–212). Göttingen: Hogrefe.
- Bromme, R. (2005). Thinking and Knowing About Knowledge: A Plea for and Critical Remarks on Psychological Research Programs on Epistemological Beliefs. In Hoffmann, M. H. G., Lenhard, J. & Seeger, F. (Eds.). *Activity and Sign – Grounding Mathematics Education*. New York: Springer.



- Brown, D. F., & Rose, T. D. (1995). Self-reported classroom impact of teachers' theories about learning and obstacles to implementation. *Action in Teacher Education*, 17(1), 20-29.
- Brunner, M., Kunter, M., Krauss, S., Baumert, J., Blum, W., Dubberke, T. et al. (2006). Welche Zusammenhänge bestehen zwischen dem fachspezifischen Professionswissen von Mathematiklehrkräften und ihrer Ausbildung sowie beruflichen Fortbildung? *Zeitschrift für Erziehungswissenschaft*, 9, 521–544.
- Calderhead, J. (1996). Teachers' beliefs and knowledge. In D. C. Berliner & R. Calfee (Eds.), *Handbook of Educational Psychology* (pp. 709- 725). New York: Macmillan.
- Cochran-Smith, M. & Zeichner, K.M. (Eds.) (2005). *Studying Teacher Education. The Report of the AERA Panel on Research and Teacher Education*. Mahwah, N.J.: Lawrence Erlbaum.
- Eurydice (2002a). *Der Lehrerberuf in Europa: Profil, Tendenzen und Anliegen, Bericht I: Lehrerausbildung und Maßnahmen für den Übergang in das Berufsleben, Allgemein bildender Sekundarbereich I*. Brüssel: Eurydice.
- Eurydice (2002b). *Der Lehrerberuf in Europa: Profil, Tendenzen und Anliegen, Bericht II: Angebot und Nachfrage, Allgemein bildender Sekundarbereich I*. Brüssel: Eurydice.
- Eurydice (2003). *Der Lehrerberuf in Europa: Profil, Tendenzen und Anliegen. Bericht III: Beschäftigungsbedingungen und Gehälter, Allgemein bildender Sekundarbereich I*. Brüssel: Eurydice.
- Eurydice (2004). *Der Lehrerberuf in Europa: Profil, Tendenzen und Anliegen, Bericht IV: Die Attraktivität des Lehrerberufs im 21. Jahrhundert, Allgemein bildender Sekundarbereich I*. Brüssel: Eurydice.
- Ferrini-Mundy, J., Schmidt, W. H., Bates, P., Joyner, T., Leroi, G. & Wigent, Ch. (2006b): *Knowing Mathematics: What We Can Learn from Teachers*. PROM/SE Research Report Vol. 2. East Lansing: Michigan State University.
- Freudenthal, H. (1983). *Didactical phenomenology of mathematical structures*. Dordrecht: D. Reidel.
- Grigutsch, S., Raatz, U., & Törner, G. (1998). Einstellungen gegenüber Mathematik bei Mathematiklehrern. *Journal für Mathematik-Didaktik*, 19, 3-45.
- Helmke, A. (2004). *Unterrichtsqualität: Erfassen, Bewerten, Verbessern* (3. Ed.). Seelze: Kallmeyersche Verlagsbuchhandlung.
- Hiebert, J., Gallimore, R., Garnier, H., Givvin, K.B., Hollingsworth, H., Jacobs, J. et al. (2003). *Teaching mathematics in seven countries: Results from the TIMSS 1999 video study* (NCES 2003-013). Washington, DC: US Department of Education.
- Hill, H. C., Ball, D. L., & Schilling, S. (2004). Developing Measures of Teachers' Mathematics Knowledge for Teaching. *Elementary School Journal*, 105(1), 11-30.
- Hill, H. C., Rowan, B., & Ball, D. L. (2005). Effects of Teachers' Mathematical Knowledge for Teaching on Student Achievement. *American Education Research Journal*, 42(2), 371-406.

- Hill, H. C. (2007). Mathematical Knowledge of Middle School Teachers. Implications for the No Child Left Behind Policy Initiative. *Educational Evaluation and Policy Analysis* 29 (2), 95-114.
- Hofer, B. K. & P. R. Pintrich (2002), *Personal Epistemology: The Psychology of Beliefs about Knowledge and Knowing*. Mahwah, NJ: Erlbaum.
- Houston, R.W. (Eds.) (1990). *Handbook of research on teacher education*. A project of the Association of Teacher Educators. New York: Macmillan
- INTASC. (1995). *Model Standards in Mathematics for Beginning Teacher Licensing and Development: A Resource for State Dialogue*. Washington D.C.: Interstate New Teacher Assessment & Support Consortium.
- Kaiser, G. (1999). *Unterrichtswirklichkeit in England und Deutschland. Vergleichende Untersuchungen am Beispiel des Mathematikunterrichts*. Weinheim: Deutscher Studien Verlag.
- Klieme, E., Schümer, G. & Knoll, S. (2001). *Mathematikunterricht in der Sekundarstufe I. „Aufgabekultur“ und Unterrichtsgestaltung*. In Bundesministerium für Bildung und Forschung (Eds.). *TIMSS – Impulse für Schule und Unterricht* (pp. 43-57). Bonn: BMBF.
- Krauthausen, G. & Scherer, P. (2007). *Einführung in die Mathematikdidaktik*. München: Elsevier.
- Kunter, M., Dubberke, T., Baumert, J., Blum, W., Brunner, M., Jordan, A. et al. (2006). *Mathematikunterricht in den PISA-Klassen 2004. Rahmenbedingungen, Formen und Lehr-Lernprozesse*. In M. Prenzel, J. Baumert, W. Blum, R. Lehmann, D. Leutner, M. Neubrand et al. (Eds.), *PISA 2003. Untersuchungen zur Kompetenzentwicklung im Verlaufe eines Schuljahres* (pp. 161–194). Münster: Waxmann.
- Leder, G. C., Pekhonen, E. & Törner, G. (Eds.) (2002): *Beliefs. A Hidden Variable in Mathematics Education?* Dordrecht: Kluwer
- Leinhardt, G. & Greeno, J. (1986): *The cognitive skill of teaching*. *Journal of Educational Psychology*, 78, 75-95.
- Leithwood, K., Edge, K., & Jantzi, D. (1999). *Educational Accountability: The State of the Art*. Gutersloh, Germany: Bertelsmann Foundation Publishers.
- Lerman, S. (2001). *A review of research perspectives on mathematics teacher education*. In F.-L. Lin & T. J. Cooney (Eds.), *Making Sense of Mathematics Teacher Education* (pp. 33–52). Dordrecht: Kluwer.
- Leung, F.K.S. (1995). *The Mathematics Classroom in Beijing, Hong Kong and London*. *Educational Studies in Mathematics*, 29, 297–325.
- Mullis, I.V.S., Martin, M.O., Gonzalez, E.J. & Chrostowski, S.J. (2004). *Findings from IEA's Trends in International Mathematics and Science Study at the Fourth and Eighth Grades*. Chestnut Hill, MA: Boston College.
- Murray, F. B. (2000). *The Role of Accreditation Reform in Teacher Education*. *Educational Policy*, 14(1), 40-59.

- National Commission on Teaching & America's Future. (1996). *What Matters Most: Teaching for America's Future*. Report of the National Commission on Teaching & America's Future. New York: Author.
- NCATE. (2000). *Professional Standards for the Accreditation of Schools, Colleges, and Departments of Education*. Washington D.C.: National Council for Accreditation of Teacher Education.
- Nespor, J. (1987). The role of beliefs in the practice of teaching. *Journal of Curriculum Studies*, 19(4), 317-328.
- Organisation für wirtschaftliche Zusammenarbeit und Entwicklung (2004b). *Teachers Matter: Attracting, Developing and Retaining Effective Teachers*. Synthesis Report: Pre-Publication Draft (18.11.2004). Paris: OECD.
- Peterson, P. L., Fennema, E., Carpenter, Th. P. & Loef, M. (1989). Teachers' pedagogical content beliefs in mathematics. *Cognition and Instruction* 6 (1), 1-40.
- PROM/SE. (2006). *Knowing Mathematics: What We Can Learn from Teachers* (Research Report Vol. 2). East Lansing: Michigan State University.
- Richardson, V. (1996). The role of attitudes and beliefs in learning to teach. In Sikula, J., Buttery, T. & Guyton, E. (Eds.): *Handbook of Research on Teacher Education* (pp. 102-119). New York: Macmillan 2. Aufl.
- Schmidt, W. H., Jorde, D., Cogan, L. S., Barrier, E., Gonzalo, I., Moser, U., et al. (1996). *Characterizing Pedagogical Flow: An Investigation of Mathematics and Science Teaching in Six Countries*. New York: Springer.
- Schmidt, W. H., McKnight, C.C., Valverde, G.A., Houang, R.T. & Wiley, D.E (1997). *Many Visions, Many Aims: A Cross-National Investigation of Curricular Intentions in School Mathematics*. Dordrecht: Kluwer.
- Schmidt, W. H., McKnight, C. C., Houang, R. T., Wang, H., Wiley, D., Cogan, L. S., et al. (2001). *Why Schools Matter: A Cross-National Comparison of Curriculum and Learning*. San Francisco, CA: Jossey-Bass.
- Schmidt, W. H., Ferrini-Mundy, J., Houang, R. T. & Cogan, L. S. (2006). *Curriculum insights from PROM/SE*. East Lansing: Michigan State University.
- Schwille, J., & Dembele, M. *Global Perspectives on Teacher Learning: Improving Policy and Practice*. Fundamentals of Educational Planning Series. Paris: International Institute for Educational Planning (IIEP), 2007.
- Short, R. J., & Short, P. M. (1989). Teacher beliefs, perceptions of behavior problems and intervention preferences. *Journal of Social Studies Research*, 13(2), 28-33.
- Shulman, L.S. (1985). Paradigms and research programs in the study of teaching: A contemporary perspective. In M.C. Wittrock (Ed.), *Handbook of Research on Teaching*, 3rd edition (pp 3-36). New York: Macmillan.
- Sikula, J., Buttery, Th. & Guyton, E. (Eds.) (1996). *Handbook of research on teacher education* (2. Ed.). New York: Macmillan.

- Staub, F. C., & Stern, E. (2002). The nature of teachers' pedagogical content belief matters for students' achievement gains: Quasiexperimental evidence from elementary mathematics. *Journal of Educational Psychology*, 94, 344-355.
- Tatto, M.T., Nielsen, H.D., Cummings, W.C., Kularatna, N.G. & Dharmadasa, D.H. (1993). Comparing the effectiveness and costs of different approaches for educating primary school teachers in Sri Lanka, *Teaching and Teacher Education*, 9 (1), 41-64.
- Tatto, M.T., Swille, J., Senk, Sh., Schmidt, W.H., Ingvarson, L., Rowley, G. & Peck, R. (2004). IEA Teacher Education Study in Mathematics (TEDS-M). Conceptual Framework. Unpublished manuscript.
- Thompson, A. G. (1992). Teachers' beliefs and conceptions: A synthesis of research. In D. A. Grouws (Ed.), *Handbook of Research on Mathematics Teaching and Learning* (pp. 127-146). New York: Macmillan.
- Valverde, G.A. & Schmidt, W. H. (2000). Greater Expectations: Learning from other nations in the quest for "world-class standards" in US school mathematics and science. *Journal of Curriculum Studies*, 32(5), 651-687.
- Vollrath, H.-J. (2001). *Grundlagen des Mathematikunterrichts in der Sekundarstufe*. Heidelberg: Spektrum Akademischer Verlag.
- Weinert, F.E. (1999). *Konzepte der Kompetenz. Gutachten zum OECD-Projekt „Definition and Selection of Competencies: Theoretical and Conceptual Foundations (DeSeCo)“*. Neuchatel, Schweiz: Bundesamt für Statistik.
- Weinert, F.E. (2001). Concept of Competence: A Conceptual Clarification. In D.S. Rychen & L.H. Salganik (Eds.), *Defining and Selecting Key Competencies* (pp. 45-66). Göttingen: Hogrefe.

## Mathematics Teaching in the 21st Century

Center for Research in Mathematics and Science Education

Michigan State University

236 Erickson Hall

East Lansing, MI 48824

(517) 353-4884 phone, (517) 432-0132 fax

**MICHIGAN STATE**  
**UNIVERSITY**



Additional copies of this report can be downloaded at <http://usteds.msu.edu>